A Revisit to Timing/Location Games with Directional Constraints

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Abstract

The research purpose of this paper is to show that an equilibrium exists in the timing/location game with directional constraints, as based on Cancian et al. (1995), if firms are unable to precommit to their timing of entry at the beginning of the game. In equilibrium, all firms sequentially enter the market, while the timing differentiation between any two consecutive entry patterns and the payoffs of all firms are equalized.

Keywords: Timing Game, Directional Constraint, Preemption Game, Precommitment Game

JEL Classification: D21, L13, R10

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Received November 6, 2018; revised January 28, 2019; accepted May 17, 2019.

經濟研究 (Taipei Economic Inquiry), 56:2 (2020), 223-241。

臺北大學經濟學系出版

1. Introduction

Cancian et al. (1995) carry out a pioneering work on a timing game with directional constraints by analyzing a TV broadcasting game whereby each television network chooses the broadcast time of its evening news to maximize its audience. Assuming the times at which viewers arrive home are continuously distributed on a finite interval and viewers watch the first newscast shown after they return home, Cancian et al. (1995) demonstrate that there is no Nash equilibrium in a pure strategy if there are at least two broadcasters. The absence of an equilibrium is because each television network will broadcast as late as possible, but still earlier than its competitors. This rules out the possibility of any strategy pair becoming a Nash equilibrium.

The Cancian et al. (1995) model also represents a location game with exogenously fixed prices and directional constraints, meaning a consumer can move in only one direction to buy products in a Hotelling (1929) model. It is the extreme case with asymmetric transportation costs, implying that moving in one direction is impossible. Since then, a rich and diverse literature on timing/location games with directional constraints has emerged. For instance, Lai (2001) expands the Cancian et al. (1995) model by employing a sequential choice of locations instead of simultaneous choices of locations. Sun (2010) is the first to add directional constraints in a spatial Cournot model, which means a firm can transport products in only one direction, in order to analyze firms' location competition in a circular-city model. Sun (2014) investigates the role played by the directional constraint in a linear-city Cournot model, showing that spatial Cournot competition with directional delivery constraints yields a richer set of spatial configurations.

This paper investigates the same timing game as Cancian et al. (1995), but with different information structures, presenting that if the firms are unable to precommit to their timing of entry at the beginning of the game, then the game possesses a unique equilibrium outcome.

One property of this outcome is that firms sequentially enter the market and that the timing differentiation between any two consecutive entry patterns and the payoffs of all firms are equalized. To the best of our knowledge, the related literature has made no attempt to directly deal with the pure timing/location decisions and to overcome the non-existence problem of equilibrium in such a game. We particularly emphasize that this current study does not incorporate any asymmetry into the Cancian et al. (1995) model and instead obtains the endogenous sequential entry.[1]

The intuition behind this is straightforward. In the game a firm can always guarantee itself of getting at least $1/n$ (i.e., the average payoff). On the other hand, the summed payoffs of all firms at most are equal to 1. As Cancian et al. (1995) point out, a firm has an incentive to delay entry if it can commit to its early entrance. However, in the preemption game the latter firms can profitably deviate to enter the market before the early firm does. Anticipating the preemption threat by the other firms, a firm enters the market once it can guarantee itself the average payoff. Consequently, the equilibrium outcome has the following properties: (i) equal distance patterns occur; (ii) all firms receive the same payoff.

This study is indeed a generalization/extension of Cancian et al. (1995) by examining two standard timing games in the related literature: (i) a precommitment game; (ii) a preemption game. Reinganum (1981a) assumes unobservable actions between duopoly firms, and that they can precommit to their timing of entry at the beginning of the game, the so-called precommitment game, thus showing that the model has a unique Nash equilibrium, in which the equilibrium involves sequential entry with higher payoffs for the first mover. The precommitment game captures the idea that the costs of

[1] The term endogenous sequential entry means that sequential entry is an equilibrium outcome of a model. On the other hand, the term exogenous sequential entry in the following means that the model (exogenously) assumes that firms choose their respective timing of entry in order.

altering the entry plans are prohibitively high or there are infinitely long information lags, corresponding to the so-called open-loop information structure.[2]

Fudenberg and Tirole (1985) preemption game conversely assumes firms can observe their rivals' entry, but they cannot credibly commit to maintain their timing of entry. This could occur, because the costs of altering entry plans are not significant or the information lags are negligible, thus referring to the so-called closed-loop information structure. They show that the potential early-mover advantage stimulates preemption, until payoffs are equalized for the firms.[3]

We note that the related literature of timing games without directional constraints, such as Argenziano and Schmidt-Dengler (2013, 2014), assumes that consumers repeatedly purchase at each instance and that their preferences on the timing of purchase are homogeneous. In this case, a firm receives a flow profit after its timing of entry into a new market. On the other hand, in timing games with directional constraints, such as Cancian et al. (1995), it is assumed that consumers' preferences on the timing of purchase are heterogeneous and directional in the sense that each consumer has a most preferred timing for a purchase as well as for purchases that start after his preferred timing of purchase. In this case, a firm receives a flow profit before its timing of entry.

The rest of this paper is organized as follows. Section 2 presents a timing/location game with directional constraints. Section 3 examines the game when firms are able and unable to precommit to their respective timing of entry at the beginning of the game, respectively. Section 4 offers the conclusion.

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2 For subsequent applications of a precommitment game, such as Reinganum (1981b), Götz (2000), Lambertini (2002), Ebina et al. (2015) and Sun (2018).

3 For subsequent applications of a preemption game, such as Riordan (1992), Hendricks (1992), Dutta et al. (1995) and Argenziano and Schmidt-Dengler (2013, 2014).

2. The Model

We assume time, denoted by t, is continuous and investigate a timing game with directional constraints, Γ, in which $n \geq 2$ payoff-maximization firms choose their respective timing of entry, $t_i \in [0,1]$, for $i \in N = \{1, 2, ..., n\}$. Define the outcome of the game as the order of entry times $(T^1, T^2, ..., T^n)$, where $0 \leq T^j \leq T^{j+1} \leq 1$ for $j = 1, 2, ..., n-1$. If the j^{th} and $(j+k)^{\text{th}}$ entries, for $k \in \{1, 2, ..., n-1\}$, occur at the same time (i.e. $T^j = T^{j+1} = \cdots = T^{j+k}$), then we say that the $(k+1)$ firms are clustered. If firm i is the j^{th} entrant, then its payoff is:

$$V_i^j(T^1,\ T^2,\ ...,T^n) = \begin{cases} T^j - T^{j-1} & \text{if firm } i \text{ is not clustered with other firms} \\ \dfrac{T^j - \max\left\{\{T^0, T^1, ..., T^{j-1}\} \setminus \{T^j\}\right\}}{k+1} & \text{if firm } i \text{ is clustered with other } k \text{ firms,} \end{cases} \tag{1}$$

where $T^0 \equiv 0$, and $X \setminus Y$ is denoted as the set $\{x : x \in X \text{ but } x \notin Y\}$. We restrict our attention to pure strategies and explore the equilibrium outcomes when the firms are both able and unable to precommit to their timing of entry, respectively.

In the precommitment game a pure strategy for firm i is a scalar $t_i \in [0,1]$, and the equilibrium concept is a Nash equilibrium. The natural equilibrium concept for a preemption game is the subgame perfect Nash equilibrium (SPNE hereafter). Following Simon and Stinchcombe (1989), we model strategies in a timing game with continuous time by assuming that time is continuous in the sense of "discrete but with a grid that is infinitely fine". At any time $t \in [0,1]$, each firm has two actions available, "wait" and "entry", conditional on the history of the game.

As is generally acknowledged, in continuous-time modeling there is no completely satisfactory way to treat joint entry. Following Dutta et al.

(1995), Hoppe and Lehmann-Grube (2005), and Argenziano and Schmidt-Dengler (2013, 2014), we introduce a randomization device in order to rule out the possibility that simultaneous entries occur as a consequence of a coordination failure, which means ex-post, each firm would regret having entered the market. In particular, we assume that if any k firms attempt to enter at any time t, then only one of them succeeds in doing so, each with probability $1/k$, while the other firms become the followers and may postpone their entry.

We use the case of $n=2$ to illustrate how the randomization device works. Consider the case in which each firm would like to enter the market at a given time, say $t=x=0.6$, given the opponent firm does not enter the market. Suppose that both firms try to enter the market at time $t=x=0.6$. In the absence of a randomization device, they both would be successful and receive a payoff $x/2=0.3$, which constitutes a coordination failure: ex-post, each firm would regret having entered the market. With the introduction of a randomization device instead, only one of them successfully enters the market, and there is no coordination failure.

Such a randomization device can be interpreted as a consequence of a capacity constraint. If there is a technology adopted with limited capacity, then the firm that adopts this technology first is the one that successfully enters the market. It can also be interpreted as a consequence of an institutional constraint and the entry itself occurs randomly. For instance, a receiving unit may randomly select one of two application attempts.

We note that if the timeline is interpreted as a geographical line, say a linear waterway, then the above timing game with directional constraints can be thought of as a location game with directional constraints. In particular, suppose there is a unit-length linear waterway with an amount of evenly distributed fish, which is normalized to one, such as Figure 1 shows.[4]

4 Lai (2001) first proposes such a fish-catching game.

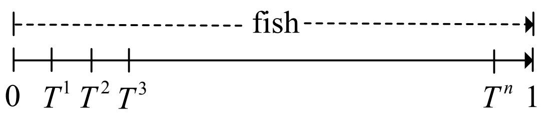

Figure 1 The Fish-Catching Game

All fish swim from the left endpoint of the linear waterway to the right endpoint of the linear waterway, which may be because there is bait dropped at the right endpoint or because the left side is upstream and the right side is downstream. When the game starts, there are n fishermen simultaneously sailing their respective n boats from the left endpoint of the linear waterway to the right endpoint of the linear waterway. All the fishermen can choose their respective location to stop and use a net to catch the fish. In such a set-up, the problem we are interested in is as follows: where to cast a net to catch the most fish for each of the fishermen if they are able and unable to commit to their respective location of casting a net at the beginning of the game.

By a similar argument, one can think of this model like a bus scheduling game. Suppose the times at which commuters reach a bus stop to wait for a bus are uniformly distributed on a finite interval and each commuter takes the first arrived bus after he reaches the bus stop. In this case, the problem we are interested in is as follows: how competing bus companies schedule their arrival times (or departure times) at a bus stop so as to maximize the number of customers.

3. The Analysis

For the case of the precommitment game, Cancian's et al. (1995) argument is replicated in that a firm's incentive to delay entry leads to no presence of an equilibrium.[5]

[5] We note in the precommitment game that Cancian et al. (1995) originally assume that firms that enter at the same time will share the payoff evenly. Although we introduce the assumption of a randomization device to rule out coordination failures in the preemption game, the non-existence result in the precommitment game still holds under such an assumption by similar logic in Cancian et al. (1995).

Result 1.

If the firms are able to precommit to their respective timing of entry at the beginning of the game, then no Nash equilibrium exists. (Cancian et al., 1995)

In the following we first treat the problem of the existence of an equilibrium in the preemption game.

Lemma 1.

In the preemption game the following strategy for each firm i, for $i=1,2,...,n,$ *constitutes a SPNE:*

- *At any* $t<1/n$, *firm i plays "wait".*
- *At any* $t\geq 1/n$, *given no previous entry, firm i plays "entry".*
- *At any* $t\geq 1/n$, *given* j *firms enter and the last entry occurs at* $t=x$, *for* $j\in\{1,2,...,n-2\}$, *firm* i *plays "wait" if* $t<x+(1-x)/(n-j)$ *and plays "entry" if* $t\geq x+(1-x)/(n-j)$.
- *At any* $t\geq 1/n$, *given* $j=n-1$ *firms enter, firm i plays "wait" if* $t<1$ *and plays "entry" if* $t=1$.

Proof. We use backward induction to show that the above strategies constitute a SPNE. At $t\geq 1/n$ given $j=n-1$ firms have entered the market, the remaining firm's best response by entering at $t=1$ is immediate. We next consider decision nodes with $t\geq 1/n$ and two active firms (i.e. $j=n-2$). First, at $t\geq(1+x)/2$, they both play "entry" from these strategies. By assumption, only one of them succeeds and the game enters a subgame with one active firm. This firm enters at $t=1$, and so both firms receive expected payoff $(1-x)/2$. Deviating and playing "wait" denote receiving a payoff smaller than $(1-x)/2$ if $t>(1+x)/2$ or receiving a payoff equal to $(1-x)/2$ if $t=(1+x)/2$ for the deviator. Second, at $t<(1+x)/2$, they both play "wait" from these strategies and play "entry" if $t\geq(1+x)/2$. As we prove above, both firms receive expected payoff $(1-x)/2$. Deviating and playing "entry" imply receiving a payoff smaller than $(1-x)/2$ for the deviator. Thus, no firm has an incentive to deviate from these strategies.

We next consider decision nodes with $t \geq 1/n$ and three active firms ($j = n-3$). First, at $t \geq x + (1-x)/3 = (1+2x)/3$, they all play "entry" from these strategies. By assumption, only one of them succeeds, and the game enters a subgame with two active firms. As we prove above, the two active firms then enter at $t = (2+x)/3$ and $t = 1$, respectively. Therefore, all three firms receive expected payoff $(1-x)/3$. Deviating and playing "wait" mean receiving a payoff of no more than $(1-x)/3$ for the deviator.

Second, at $t < x + (1-x)/3$, they all play "wait" from these strategies. As we prove above, all three firms receive expected payoff $(1-x)/3$ from the strategies. Deviating and playing "entry" show a payoff smaller than $(1-x)/3$ for the deviator. Thus, no firm has an incentive to deviate from these strategies. Repeating the same argument for $j = n-4, n-5, \ldots, 1$, the recursive algorithm can be used to show that at any decision node with $t \geq 1/n$ and any number $(n-j)$ of active firms, firm i plays "wait" if $t < x + (1-x)/(n-j)$ and plays "entry" if $t \geq x + (1-x)/(n-j)$.

We finally consider decision nodes with $t \geq 1/n$ and n active firms. First, at $t = x \geq 1/n$, they all play "entry" from these strategies. By assumption, only one of them succeeds, and the game enters a subgame with $(n-1)$ active firms. As we prove above, the $(n-1)$ active firms then sequentially enter at $(2/n, 3/n, \ldots, 1)$. Therefore, all the n firms receive expected payoff $1/n$. Deviating and playing "wait" denote receiving an expected payoff of no more than $1/n$ for the deviator.

Second, at $t < 1/n$, they all play "wait" from these strategies. As we prove above, all the n firms receive expected payoff $1/n$ from the strategies. Deviating and playing "entry" mean receiving a payoff smaller than $1/n$ for the deviator. Thus, no firm has an incentive to deviate from these strategies. In sum, the described strategies are best responses to each other and constitute a Nash equilibrium in every subgame $\gamma \in \Gamma$.

□

Proposition 1 shows the uniqueness of the equilibrium outcome.

Proposition 1.

(i) If the firms are unable to precommit to their respective timing of entry at the beginning of the game, then there exists a unique SPNE outcome, up to a permutation of firms, such that $T^j = j/n$, *for* $j = 1, 2, ..., n$.

(ii) In equilibrium firms sequentially enter the market, and the timing differentiation between any two consecutive entry patterns and the payoffs of all firms are equalized.

Proof. According to the strategy for each firm identified in Lemma 1, a firm can always guarantee itself of getting at least $1/n$, no matter what strategies the other firms choose. Since the summed payoffs of all firms must at most be equal to 1, each firm receives payoff $1/n$ in equilibrium. We claim the first entry occurs at $T^1 = 1/n$. By contradiction, if the first entry occurs at $T^1 = x < 1/n$, then this firm must have an incentive to deviate to "wait", given its opponents' strategies. If the first entry occurs at $T^1 = x > 1/n$, then one of the other firms must have an incentive to deviate to "entry" at $t < x$, given its opponents' strategies. By induction, the second entry must occur at $T^2 = 2/n$, and $(T^1 = 1/n,\ T^2 = 2/n, ...,\ T^n = 1)$ is the unique equilibrium outcome. Under the assumption of a randomization device, the equilibrium strategies identified above have the properties illustrated in Proposition 1 (ii).

□

We note that the assumption of a randomization device effectively rules out the case of clustering in such a timing game with directional constraints. We use the two-firm case ($n = 2$) to obtain some intuition behind the results. Appendix illustrates the derivation of Proposition 1 with two firms ($n = 2$) in greater detail. The recursive algorithm can be used to construct the equilibrium with a general n-firm game.

When there are two firms, firm 1 can guarantee a payoff greater than or equal to $1/2$, and thus it plays "wait" if $t < 1/2$. Given that firm 2 enters at $t = 1$, firm 1 has an incentive to delay entry at $t \in [1/2, 1)$. However, in the preemption game, firm 1 cannot commit to be the first entrant, and firm 2 can profitably deviate to enter at $t > 1/2$ if firm 1 plays "wait". Anticipating firm 2's best response, firm 1 enters the market once it can guarantee itself the average payoff (i.e. $t_1 = 1/2$). The same argument applies to show that firm 2 enters the market once it can guarantee itself the average payoff (i.e. $t_2 = 1/2$). Under the assumption of a randomization device, the first entry occurs at $T^1 = 1/2$, and the second entry occurs at $T^2 = 1$.

We note the assumption of the space being linear is not necessary for derivation of our main results of equal distance patterns and payoff equalization, provided customers are distributed uniformly on the interval. On the other hand, if we relax the assumption of uniformly distributed customers, then the result of payoff equalization is still valid, but unequal distance locations may emerge in equilibrium.

We note that Cancian et al. (1995) claim in a Stackelberg (exogenously sequential-entry) model with two firms that the only equilibrium is firms dividing the market into equalized shares.[6] This current study, in contrast, does not incorporate any asymmetry into the model and obtains the endogenous sequential entry. We also note that some efforts in the related literature of location theory have attempted to incorporate additional price/quantity competition into Cancian's et al.

6 We note Cancian et al. (1995) do not provide a rigorous proof of such an equilibrium. Lai (2001) analyzes the competition behavior in a directional market under discontinuous time and exogenously sequential-entry, showing the existence of an equilibrium.

(1995) timing/location game and to discuss the equilibrium property. However, no attempt has been made to directly deal with the pure timing/location decisions and to bypass the non-existence problem of equilibrium in the game. Interestingly, the equal distance patterns proposed in Proposition 1 are very rare in a spatial model with Hotelling's (1929) linear market, while there are such reminiscent results in a spatial model with Salop's (1979) circular market in the literature.[7]

4. Conclusion

This paper overcomes the non-existence problem of equilibrium in the Cancian et al. (1995) model by considering two alternative timing games with different information structures. The first alternative assumes that firms are able to precommit to their respective timing of entry at the beginning of the game, which is the so-called precommitment game. On the other hand, the second alternative assumes firms are unable to precommit to their respective timing of entry at the beginning of the game, which is the so-called preemption game.

For the case of the precommitment game, Cancian et al. (1995) argument is replicated in that a firm's incentive to delay entry leads to no equilibrium existing in the model. By contrast, we show that if the firms are unable to commit to their timing of entry, then the game possesses a unique equilibrium outcome. The outcome has the following properties: (i) equal distance patterns occur; (ii) all firms receive the same payoff.

The issue of directional constraints is realistic and applicable for bus

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7 In particular, Economides (1989) derives the equal distance locations in Salop's (1979) circular market, while Brenner (2005) achieves the result of unequal distance locations in Hotelling's (1929) linear market when there are more than three firms.

scheduling times, in which customers only move forward to the next selling point. In this case, consumers wait for products to be released. For instance, consumers may wait for new products, say a new generation of smartphones, to enter the market, and competing smartphone companies decide over when to release their respective new smartphones. The issue of directional constraints is also valid for firms' location competition when consumers or firms may move in only one direction on one-way roads or highways. For instance, tourists may move on a unidirectional (linear or non-linear) recommended route to a tourist destination to buy souvenirs. In such a set-up, the problem we are interested is as follows: where to set up stalls for vendors selling souvenirs on the recommended route to a tourist destination.

Central to our finding is that Fudenberg and Tirole's (1985) result of payoff equalization is still valid under such a timing game with directional constraints. To the best of our knowledge, this study is the first to apply the concept of a preemption equilibrium to the timing game with directional constraints. It is of further interest to investigate both firms' timing and product-market competition by incorporating a price or quantity competition into the preemption game with directional constraints.

Appendix

When there are two firms, the payoff matrix in the last period, given no previous entry, is as follows:

1 \ 2	entry	wait
entry	$\left(\frac{1}{2}, \frac{1}{2}\right)$	$(1, 0)$
wait	$(0, 1)$	$(0, 0)$

Figure A1 The Payoff Matrix in the Last Period ($t=1$), Given No Previous Entry

Given the two firms do not enter the market at time $t<1$, each firm's dominant strategy is to choose "entry" in the last period ($t=1$), and each one receives a payoff $1/2$. Turning to the second-last period ($1/2<t=x<1$), given no previous entry, the dominant strategy for each firm is also to choose "entry", and each one receives a payoff $1/2$, such as the following Figure A2 shows.

1 \ 2	entry	wait
entry	$\left(\frac{1}{2}, \frac{1}{2}\right)$	$(x, 1-x)$
wait	$(1-x, x)$	$\left(\frac{1}{2}, \frac{1}{2}\right)$

Figure A2 The Payoff Matrix in the Second-Last Period ($1/2<t=x<1$), Given No Previous Entry

By induction, given no previous entry at time $t=x>1/2$, each firm's dominant strategy is to choose "entry", and each one receives a payoff $1/2$. At time $t=1/2$, choosing "entry" is the weakly dominant strategy for each firm, and choosing "entry" for both firms constitutes a Nash equilibrium in this subgame. By similar arguments, it is easy to see that given neither of the two firms enters the market at time $t = x < 1/2$, choosing "wait" is each firm's dominant strategy. Therefore, the strategy combination illustrated in Lemma 1 constitutes a Nash equilibrium in every subgame. Under the assumption of a randomization device, one firm enters the market at $t = 1/2$, and the other enters the market at $t = 1$ in equilibrium, and each firm receives a payoff $1/2$.

We also note that at time $t = 1/2$, one firm choosing "entry" and the other choosing "wait" constitute a Nash equilibrium in this subgame. In addition, choosing "wait" for both firms also constitutes a Nash equilibrium. The two firms enter the next period ($t = 1/2 + \varepsilon$, for $\varepsilon \to 0^+$). As mentioned above, given no previous entry at time $t > 1/2$, choosing "entry" is each firm's dominant strategy. Under the assumption of a randomization device, one firm enters the market at $t = \lim_{\varepsilon \to 0^+} 1/2 + \varepsilon = 1/2$, and the other one enters the market at $t = 1$ in equilibrium. In sum, as shown in Proposition 1, the unique equilibrium outcome is that the first entry occurs at $T^1 = 1/2$, the second entry occurs at $T^2 = 1$, and each firm receives a payoff $1/2$.

References

Argenziano, R. and P. Schmidt-Dengler (2013), "Competition, Timing of Entry and Welfare in a Preemption Game," *Economics Letters*, 120:3, 509-512.

Argenziano, R. and P. Schmidt-Dengler (2014), "Clustering in N-Player Preemption Games," *Journal of the European Economic Association*, 12:2, 368-396.

Brenner, S. (2005), "Hotelling Games with Three, Four, and More Players," *Journal of Regional Science*, 45:4, 851-864.

Cancian, M., A. Bills and T. Bergstrom (1995), "Hotelling Location Problems with Directional Constraints: An Application to Television News Scheduling," *The Journal of Industrial Economics*, 43:1, 121-124.

Dutta, P. K., S. Lach and A. Rustichini (1995), "Better Late Than Early: Vertical Differentiation in the Adoption of a New Technology," *Journal of Economics & Management Strategy*, 4:4, 563-589.

Ebina, T., N. Matsushima and D. Shimizu (2015), "Product Differentiation and Entry Timing in a Continuous Time Spatial Competition Model," *European Journal of Operational Research*, 247:3, 904-913.

Economides, N. (1989), "Symmetric Equilibrium Existence and Optimality in Differentiated Product Markets," *Journal of Economic Theory*, 47:1, 178-194.

Fudenberg, D. and J. Tirole (1985), "Preemption and Rent Equalization in the Adoption of New Technology," *The Review of Economic Studies*, 52:3, 383-401.

Götz, G. (2000), "Strategic Timing of Adoption of New Technologies under Uncertainty: A Note," *International Journal of Industrial*

Organization, 18:2, 369-379.

Hendricks, K. (1992), "Reputation in the Adoption of a New Technology," *International Journal of Industrial Organization*, 10:4, 663-677.

Hoppe, H. C. and U. Lehmann-Grube (2005), "Innovation Timing Games: A General Framework with Applications," *The Journal of Economic Theory*, 121:1, 30-50.

Hotelling, H. (1929), "Stability in Competition," *The Economic Journal*, 39, 41-57.

Lai, F. C. (2001), "Sequential Locations in Directional Markets," *Regional Science and Urban Economics*, 31:5, 535-546.

Lambertini, L. (2002), "Equilibrium Locations in a Spatial Model with Sequential Entry in Real Time," *Regional Science and Urban Economics*, 32:1, 47-58.

Reinganum, J. F. (1981a), "On the Diffusion of New Technology: A Game Theoretic Approach," *The Review of Economic Studies*, 48:3, 395-405.

Reinganum, J. F. (1981b), "Market Structure and the Diffusion of New Technology," *The Bell Journal of Economics*, 12:2, 618-624.

Riordan, M. H. (1992), "Regulation and Preemptive Technology Adoption," *The RAND Journal of Economics*, 23:3, 334-349.

Salop, S. C. (1979), "Monopolistic Competition with Outside Goods," *The Bell Journal of Economics*, 10:1, 141-156.

Simon, L. K. and M. B. Stinchcombe (1989), "Extensive Form Games in Continuous Time: Pure Strategies," *Econometrica*, 57:5, 1171-1214.

Sun, C. H. (2010), "Spatial Cournot Competition in a Circular City with Directional Delivery Constraints," *The Annals of Regional Science*, 45:2, 273-289.

Sun, C. H. (2014), "Spatial Cournot Competition in a Linear City with Directional Delivery Constraints," *The Japanese Economic*

Review, 65:3, 344-374.

Sun, C. H. (2018), "Timing of Entry and Location/Product Differentiation," *International Journal of Economic Theory*, 14:2, 179-200.

再探方向性限制下的時點（區位）賽局

孫嘉宏*

摘　要

本文證明，若廠商無法在賽局期初，事先承諾其進入市場的時點，則 Cancian et al. (1995) 方向性限制下的時點（區位）賽局均衡存在。廠商均衡時依序進入市場，且任兩家依序進入市場的廠商之時點差異均等，而所有廠商均衡利潤相等。

關鍵詞：時點賽局、方向性限制、先發制人賽局、事先承諾賽局
JEL 分類代號：D21, L13, R10

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投稿日期：民國 107 年 11 月 6 日；修訂日期：民國 108 年 1 月 28 日；接受日期：民國 108 年 5 月 17 日。