

Long-Run Effects of Inflation Under Calvo Staggered Price Contract

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Abstract

Under Calvo staggered price contracts, there are two types of production inefficiency associated with long-run inflation. One is the average markup distortion that holds back the final production level from its social optimal level. The other is the price variation distortion due to the price erosion from inflation that inefficiently results in more social resources producing the same level of output. The standard Calvo model shows that inflation monotonically enhances both distortions, and therefore a positive long-run inflation target is not desired. However, when the price adjustment frequency is endogenous, the price adjustment speed increases with inflation. Without increasing much price adjustment frequency, minor inflation can erode vintage markups and pull down the average markup, with the output level pushing closer to the social optimum. This partially corrects the average markup distortion. When the inflation rate is low, this gain can outweigh the loss from price variation distortion. We also find that the more competitiveness there is among producers, the lower the optimal long-run inflation target shall be.

Keywords: Calvo Model, Long-Run Inflation Target

JEL Classification: E31, E50, E52

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Received 24 September 2012; revised 8 April 2013; accepted 31 March 2014.

1. Introduction

In the research regarding monetary policy, it is common to embody the price rigidity in the model. The ways of modeling the rigidity can be divided into the state-dependent models such as Caplin and Leahy (1991), Dotsey et al. (1999) and Khan et al. (2003) and the time-dependent models, such as Taylor (1980) and Calvo (1983). Tracking the dynamic outcome in the former is more challenging. As a result, the time-dependent models are often used to study the dynamic outcome under various monetary policy rules.

The standard approach that adopts the time-dependent contract assumes a stable long-run inflation level and a stable contract format (i.e. an exogenous price adjustment frequency). The dynamic analysis are often based on a local linearization around the zero inflation level. This approach leaves no room for the study of the long-run inflation level in their research since all the variables are transformed to their deviation from their trends. The questions for optimal monetary policy rule can only circle around how the monetary authority should respond to inflation deviations (from its long-run target) and to output deviations (from its potential level). The optimal long-run inflation rate is usually ignored. The other reason for the long-run inflation target being widely missed in the literature also has to do with the long-run monetary neutrality argument in the literature. Even though such a long-run proposition is widely recognized by the monetary economists, its empirical assessments do not give an uniform answer (Fisher and Seater, 1993; Geweke, 1986; King and Watson, 1997).

In this paper we study the long-run inflation and efficiency trade-off under the economy of the time-dependent sticky price contract,

namely Calvo contract (Calvo, 1983). For Calvo contract, producers draw a random signal in each period. The signal determines whether the producers are able to re-set their prices. We also add-in an adjustment cost to endogenize the price adjustment frequency.

Long-run superneutrality of money does not hold in the model. When the contract is stable (i.e., fixed price adjusting probability), the real effect of inflation comes from two channels: the average markup channel and the price variation channel. Average markup channel is related to the inefficient output level that results from the monopolistic competitive market. Price variation channel is related to an inefficient production process that uses too much input resource to produce a given output level. In a standard Calvo model (where price adjustment frequency is exogenous), higher inflation induces a higher average markup as well as a larger price variation. The former pulls down output level further lower from efficiency. The latter, coming from the price erosion through inflation, motivates the final goods producers to use a more uneven level of intermediate inputs for production. With the final goods production function being convex, it means that such a production process is less efficient. Both effects have a negative impact on the final output. Consequently, we see a negative trade-off between long-run inflation and the real output.

The above contractionary result of inflation is based on the model with a stable contract. Golosov and Lucas (2007) found that the frequency of price adjustments is highly reduced in a low inflationary environment rather than in a high inflationary environment. This is consistent with other empirical findings such as Lach and Tsiddon (1992) study on the Israeli inflations of 1978-1979 and 1981-1982. Firms tend to adjust their prices more frequently when they are in a higher inflationary environment. We relax the exogenous price adjustment

frequency by introducing a fixed adjustment cost. Firms choose their price adjustment frequency that maximizes their long-run production values in the steady state. As a result, the producers adjust their prices more frequently when the long-run inflation is higher.

Price adjustment frequency affects both the average markup and the price variation magnitudes. A higher adjustment frequency pulls down the average markup as well as the price variation. The extreme case is the flexible pricing environment where the average markup is equal to the static markup and no price variation in the long run. From both angles, a higher price adjustment frequency can improve the production efficiency. When we put inflation back to the picture, a higher inflation conditional on the price adjustment frequency decreases the production efficiency; but it increases the price adjustment speed, which increases the production efficiency. As these two effects contradict each other, the observed long-run trade-off between inflation and output become negligibly small, though not zero, compared to the standard Calvo model. This is probably why the conventional tests on the long-run neutrality of money have very low power since the true alternative is very close to the null hypothesis (Friedman, 2005; Geweke, 1986).

King and Wolman (1996) found that in a Calvo model with exogenous price adjustment frequency lowering inflation is welfare improving even in the deflationary regime. Friedman rule that promotes zero nominal interest rate targeting is supported in their result as it requires a deflationary target under positive real interest rate scenario. However, when the price adjustment frequency is endogenized, we find that decreasing inflation is not always welfare improving. It is better to preserve some minor inflation in the long run. Zero inflation is not the best choice. This is because under zero inflation the prices for

intermediate goods are still higher than their marginal cost. The average markup distortion still exists; a little bit of inflation can actually bring down the markup. Though this will trigger a loss of production efficiency due to a larger price variation, this loss is not big when inflation is minor.

A minor inflation target can be justified only if it can bring down the average markup lower than the zero inflation target. When the price adjustment frequency is endogenous, firms will choose to never adjust their prices under zero inflation in the long run. This is to avoid the adjustment cost. If we move the inflation target a little bit higher from zero, the price adjustment frequency, though getting higher, is still low. With positive inflation, the real markups of the old vintage price items will be eroded. It brings down the average markup and promotes a more efficient output level with a trade-off of a larger price variation that decreases production efficiency. However, this loss is small when the inflation is small.

The strength of the markup effect is closely related to the magnitude of the market competitiveness. We also examine the optimal long-run inflation target level under various market competitive strength. We find that when the market power of the intermediate producers is weaker, the optimal inflation target should be lowered. To understand the reason, it is useful to think of an extreme case of a perfectly competitive market. In such an environment, there is no markup distortion under zero inflation. A higher inflation only induces price variation and reduces production efficiency. This indicates that a positive inflation target can be justified only if there is a markup distortion. When the market competition is moved toward perfect competition, the optimal inflation target has to move toward zero inflation.

This paper is divided into seven sections. Section 2 lays out the model. Section 3 defines the price index, inflation and their relationship with the zero-vintage price. Section 4 defines the steady state equilibrium. Section 5 discusses the relationship between inflation, production efficiency and markup distortion. Section 6 shows the calibration results of the models. Section 7 draws the conclusion.

2. The Model

2.1 Household

There is a representative agent whose contemporaneous utility depends on the final good consumption c and the amount of labor work N . The contemporaneous utility function is set up to be:

$$u(c, N) = \log(c) - \frac{1}{\xi} \frac{N^{\sigma+1}}{\sigma+1}.$$

Therefore, the first order condition (FOC) regarding the trade-off between today's labor and consumption provides the labor supply function:

$$N = \xi^{\frac{1}{\sigma}} \left(\frac{w}{c} \right)^{\frac{1}{\sigma}}, \quad (1)$$

where w is the real wage rate. With the consumption component set in logarithm form in the utility function, the labor supply function ensures that a permanent productivity change that shifts consumption and the wage rate proportionally leaves labor supply unchanged. This is a common setup in the growth model so as to prevent the labor supply from going beyond the time constraint.

Given the real interest rate R_{t+1} and the utility discount rate β , in equilibrium Euler equation is satisfied:

$$u_c(c_t, N_t) = \beta \mathbb{E}_t(1 + R_{t+1})u_c(c_{t+1}, N_{t+1}).$$

We study the long-run equilibrium without productivity growth. In equilibrium $c_t = c$ and the interest rate $R_t = R$ with:

$$1 + R = \frac{1}{\beta}, \quad (2)$$

which is independent of the utility functional form. The steady state interest rate solely depends on the utility discount rate.

Household real budget constraint is expressed as:

$$c_t + \Delta K_t^s + \delta K_t^s + \Delta B_t^d = w_t N_t + r_t K_t^s + \phi_t + R_t B_t^d.$$

The LHS of the budget constraint is the consumption plus savings. The RHS of the budget constraint is the income source. Saving consists of capital investment $\Delta K_t^s + \delta K_t^s$ and the bond saving ΔB_t^d . Capital investment contains both net investment $\Delta K_t^s = K_{t+1}^s - K_t^s$ and the replacement investment δK_t^s given δ represents the capital depreciation rate. The definition of $\Delta B_t^d = B_{t+1}^d - B_t^d$. For the income source, it contains the labor income $w_t N_t$, capital rental income $r_t K_t^s$ given the rental price r_t , the dividend payment ϕ_t from the profits of all firms, and the interest payment from bond holding $R_t B_t^d$. In the equilibrium, $B_t^d = 0$, $\Delta K_t^s = 0$ and $w_t N_t + r_t K_t^s + \phi_t = y_t$, where y_t is the total final output. We also consider a fixed adjustment cost A in terms of final goods that firms have to pay for a price tag change. In each period, there will be α portion of firms adjusting their prices; the society has to sacrifice αA units of final goods. Therefore, under steady state, the budget constraint can be reduced to a market clearing condition as:

$$c + \delta K^s = y - \alpha A. \quad (3)$$

2.2 Producers

There are two types of producers: final goods producers and the intermediate goods producers (intermediate producers, hereafter).

2.2.1 Final goods producers

The final goods market is perfectly competitive. The producers produce goods that can be either consumed or invested in capital by the household. Each producer faces the same production function in which the amount of final goods produced y can be expressed as:

$$y = \left(\int y_i^\rho di \right)^{\frac{1}{\rho}},$$

as an output from heterogeneous intermediate goods inputs (y_i) the structure of which will be explained in detail later. The subscript i categorize intermediate producers on a continuum of $[0,1]$. Given the intermediate good prices (P_i), the marginal cost of final goods production will be $\left(\int P_i^{-\rho/(1-\rho)} di \right)^{-(1-\rho)/\rho}$. In equilibrium, perfect market competition requires that the price of final goods:

$$P = \left(\int P_i^{\frac{-\rho}{(1-\rho)}} di \right)^{\frac{-(1-\rho)}{\rho}}. \quad (4)$$

This price represents the price index of an economy and will be the index for inflation computation in the model.

For the intermediate good i , cost minimization derives its demand:

$$y_i^d = \left(\frac{P_i}{P} \right)^{\frac{-1}{(1-\rho)}} y. \quad (5)$$

The demand function is negatively related with the real price of input i and positively related with the equilibrium output level of final goods. The parameter ρ lies between zero and one so as to guarantee

the existence and uniqueness of the optimal pricing for the monopolistic competition in the intermediate goods market.

2.2.2 Intermediate goods producers

Intermediate goods are heterogeneous. Only one producer produces each type of goods. The total population of intermediate producer is normalized to one. The production of intermediate goods requires labor input n and capital input k . For producer i , his production function for output y_i is assumed to follow a Cobb-Douglas function:

$$y_i = a n_i^\gamma k_i^{1-\gamma},$$

with $0 < \gamma < 1$ and a being the total productivity level. The labor and capital input markets are perfectly competitive. Given the real wage rate w and real capital rental price r , cost minimization suggests that the input demand ratio follows:

$$\frac{\gamma}{1-\gamma} \left(\frac{k_i}{n_i} \right) = \frac{w}{r},$$

which is the condition that the marginal productivity of input ratios must be equal to the input prices ratio. And its real marginal cost ψ_i will follow:

$$\psi_i = \frac{1}{a} \left[w \left(\frac{\gamma}{1-\gamma} \frac{r}{w} \right)^{1-\gamma} + r \left(\frac{\gamma}{1-\gamma} \frac{r}{w} \right)^{-\gamma} \right], \quad (6)$$

which is the same across all intermediate producers. We can write $\psi = \psi(w, r)$, dropping the subscript i . Combining with the production function, its demand functions for the inputs are:

$$n_i = \left(\frac{\gamma}{1-\gamma} \frac{r}{w} \right)^{1-\gamma} \frac{y_i}{a}, \quad (7)$$

$$k_i = \left(\frac{1-\gamma}{\gamma} \frac{w}{r} \right)^\gamma \frac{y_i}{a}. \quad (8)$$

From equation (7) and equation (8), we can find that $w/\gamma = [\gamma/(1-\gamma)](K/N)$ where K and N are aggregate capital and aggregate labor.¹ The RHS expression represents the MPN/MPK ratio for the production function $y^p = aN^\gamma K^{1-\gamma}$ which represents the potential production function for the final goods output. The equilibrium final goods output y might be different from the potential output y^p . We will explore this issue when we talk about production inefficiency later. However, the equality, $w/\gamma = [\gamma/(1-\gamma)](K/N)$, indicates that the input prices will not be the cause of production inefficiency.

The real profit function for producer i at time t is $\phi_{i,t}$:

$$\phi_{i,t} = \left(\frac{P_{i,t}}{P_t} - \psi_t \right) y_{i,t}.$$

Intermediate producer i maximizes his present discounted value of profit which is $\sum_{t=0}^{\infty} (1+R)^{-t} \phi_{i,t}$.² The total profit $\int \phi_{i,t} di = \phi_t$ is the dividend income source of household. For the convenience of discussion, we denote the real (relative) price $q_{i,t} \equiv P_{i,t}/P_t$.

2.2.3 Pricing problem

Intermediate producers are switching between two statuses, either setting a new price or keeping the price tag unchanged. With the possibility of status switching, it is more convenient to formulate the problem in terms of Bellman equations, and categorize intermediate producers by their price vintages. The vintage is denoted by subscript k throughout this paper.³ In the standard Calvo model, each period α

¹ We have $K = \int k_i di$ and $N = \int n_i di$.

² In the steady state equilibrium, we know from equation (2) that the interest rate discounting is equivalent to utility discounting.

³ Subscript t is for time. Subscript i is for the i -th uniquely identified intermediate producer. Subscript k is for those intermediate producers who charge a k -vintage price.

portion of intermediate producers are chosen to be able to reset their prices. This means that $\alpha(1-\alpha)^k$ is the portion of the intermediate producers who charge a k -vintage price. Let v_k denote the (real) value of a price-rigid firm who is charging a k -vintage price, and v_0 is the value of a price-flexible firm. The value functions will evolve according to:

$$v_0 = \max_{\{q_0\}} \phi_0 + \beta(1-\alpha)v_1 + \beta\alpha(v_0 - A),$$

$$v_k = \phi_k + \beta(1-\alpha)v_{k+1} + \beta\alpha(v_0 - A),$$

where q_0 is the zero-vintage (real) price, $\phi_k = \phi(q_k)$ is the profit function for the producer charging a k -vintage price $q_k \equiv P_k/P$. Since we only focus on the steady state equilibrium, the time subscript is dropped; only vintage subscript is kept, and the interest rate discounting factor is replaced with the utility discounting factor.

A marginal increase Δq_0 of the zero-vintage (real) price q_0 will increase the k -vintage price q_k by $\pi^{-k}\Delta q_0$ where $\pi \equiv P_{t+1}/P_t$ for all t is the steady state inflation level. The marginal change of q_k affects future revenue k periods after by $MRP_k = -[\rho/(1-\rho)]q_k^{-1/(1-\rho)}y$ and the future cost by $MCP_k = -\psi[1/(1-\rho)]q_k^{-1/(1-\rho)-1}y$. The higher the real price, the lower the revenue, and the lower the cost. If we take a negative sign on both MRP and MCP , then $-MRP$ represents the marginal revenue sacrifice by increasing the price and $-MCP$ represents the marginal cost saving by increasing the price. Therefore, $-MRP$ is the marginal cost of pricing and $-MCP$ is the marginal benefit of pricing. If the firm sets its real price q_k to equate both marginal trade-offs, then $q_k = \psi/\rho$ which is a $1/\rho$ markup of the real marginal cost of production ψ . In this case, ϕ_k reaches its maximal. This is the optimal pricing under a static or flexible pricing model. For the convenience of our discussion, we call $1/\rho$ the optimal static markup. The discrepancy between price and the marginal cost indicates a markup distortion. The existence of markup distortion gives a

standpoint for the monetary authority to target a positive inflation target, which will be discussed in detail later.

Under price stickiness setting, the optimal pricing of q_0 has to take into account its impact on future real price tags as $q_k = \pi^{-k}q_0$. A marginal increase of q_0 increases the k -vintage revenue by $MRP_k \times \pi^{-k}$ and increases the k -vintage cost by $MCP_k \pi^{-k}$. Optimal pricing sets the sum of the expected marginal profit of various vintage prices to zero so that:

$$(MRP_0 - MCP_0) + \sum_{k=1}^{\infty} (\beta(1-\alpha))^k (MRP_k - MCP_k) \pi^{-k} = 0. \quad (9)$$

Price adjustment cost is not in the picture there since it is sunk cost. We can rearrange the above condition and obtain:

$$\rho \left\{ 1 + \sum_{k=1}^{\infty} [\beta(1-\alpha)]^k \pi^{\frac{k\rho}{1-\rho}} \right\} = \psi q_0^{-1} \left\{ 1 + \sum_{k=1}^{\infty} [\beta(1-\alpha)]^k \pi^{\frac{k}{1-\rho}} \right\}.$$

Assuming $0 < \beta(1-\alpha)\pi^{1/(1-\rho)} < 1$ so the infinite sum converges. The optimal price will be:

$$q_0 = \frac{1}{\rho} \left\{ \frac{1 - [\beta(1-\alpha)] \pi^{\frac{\rho}{1-\rho}}}{1 - [\beta(1-\alpha)] \pi^{\frac{1}{1-\rho}}} \right\} \psi. \quad (10)$$

There are several features worth mentioning. First, under positive inflation (i.e. $\pi > 1$) the initial markup q_0/ψ is larger than the optimal static markup $1/\rho$ as $0 < \rho < 1$. This makes intuitive sense. If $q_0/\psi = 1/\rho$ which yields $MRP_0 = MCP_0$, then inflation will erode future markups $q_k/\psi < 1/\rho$ for $k > 0$. It follows that in the future $MRP_k > MCP_k$. Then the LHS of equation (9) will be greater than zero.

Firms have the incentive to set q_0/ψ higher to enjoys a higher profit stream from the future. Second, the higher the adjustment speed (i.e., a larger α) the higher the initial markup. This is because higher α means a higher chance to obtain the future profit from a vintage price. With inflation erosion, distant future (i.e., large k) has $q_k/\psi < 1/\rho$.⁴ It means $MRP_k - MCP_k > 0$. This creates an incentive for the firm to increase future q_k/ψ by increasing the initial markup. Third, the higher the inflation, the higher the initial markup. This is because higher inflation erodes future markup further. The firm has the incentive to insure against that by increasing initial markup.⁵

3. Inflation and Real Prices

When we categorize intermediate goods producers by their price vintages (using subscript k), the price index of final good price can be expressed as:

$$P_t = \left[\sum_{k=0}^{\infty} \alpha(1-\alpha)^k P_{k,t}^{\frac{-\rho}{1-\rho}} \right]^{\frac{-(1-\rho)}{\rho}}. \quad (11)$$

Diving both sides by P_t , we obtain:

$$\begin{aligned} 1 &= \left[\sum_{k=0}^{\infty} \alpha(1-\alpha)^k \left(\frac{P_{k,t}}{P_t} \right)^{\frac{-\rho}{1-\rho}} \right]^{\frac{-(1-\rho)}{\rho}} \\ &= \left[\sum_{k=0}^{\infty} \alpha(1-\alpha)^k \left(\frac{P_{k-t}}{P_t} \frac{P_{0,t-k}}{P_{t-k}} \right)^{\frac{-\rho}{1-\rho}} \right]^{\frac{-(1-\rho)}{\rho}}. \end{aligned}$$

⁴ To be exact, after $\log(\tau)/\log(\pi)$ periods, the real price tag will be lower than ψ/ρ , where $\tau = \{1 - [\beta(1-\alpha)]\pi^{\rho(1-\rho)}\} / \{1 - [\beta(1-\alpha)]\pi^{1(1-\rho)}\}$.

⁵ The mathematical implication of inflation is not as transparent as the change of α . We provide its proof in Appendix 1.

The second line uses the fact that the k -vintage price at time t is a new price at time $t-k$, i.e. $P_{k,t} = P_{0,t-k}$. In the steady state, $P_{0,t}/P_t = q_0$ a constant for all t and $P_t/P_{t-1} = \pi$ for all t as well. Therefore, in steady state equilibrium:⁶

$$1 = (1 - \alpha)\pi^{\frac{\rho}{1-\rho}} + \alpha q_0^{\frac{-\rho}{1-\rho}}.$$

We can sort out q_0 so that

$$q_0 = \left[\frac{\alpha}{1 - (1 - \alpha)\pi^{\frac{\rho}{1-\rho}}} \right]^{\frac{1-\rho}{\rho}}. \quad (12)$$

For monetary authority to target a specific inflation, it is equivalent to targeting a certain real price level q_0 as long as firms are not all price flexible. When all firms are price flexible, $\alpha = 1$. The RHS will be one. Inflation drops out of the equation. From equation (12), monetary authority can target a higher real price by targeting a higher inflation level.

4. Steady State Equilibrium Conditions

Given a steady state inflation level π , a steady state equilibrium is an allocation $\{c, N, K, y, \{y_k, n_k, k_k\}_{k=0}^{\infty}\}$ with the price mechanism $\{\{q_k\}_{k=0}^{\infty}, r, w, R\}$ that satisfies the following conditions:

- (i) q_0 is consistent with the inflation-price trade-off equation (12) and q_k evolves according to $q_k = \pi^{-k}q_0$.
- (ii) Marginal cost ψ defined as equation (6) supports the optimal pricing rule of equation (10) for q_0 .

⁶ With $q_k = q_0\pi^{-k}$, it follows that

$$\begin{aligned} 1 &= \left[\sum_{k=0}^{\infty} \alpha(1-\alpha)^k \pi^{k\rho/(1-\rho)} q_0^{-\rho/(1-\rho)} \right]^{-(1-\rho)/\rho} \\ &= \left\{ \alpha q_0^{-\rho/(1-\rho)} / \left[1 - (1-\alpha)\pi^{\rho/(1-\rho)} \right] \right\}^{-(1-\rho)/\rho}. \end{aligned}$$

- (iii) Under y and q_k , the y_k amount of k -vintage intermediate goods is produced according to the demand function in equation (5).
- (iv) Under y_k the labor demand n_k and capital demand k_k are defined by equation (7) and equation (8) respectively.
- (v) Given c and w , labor supply N satisfy the FOC of equation (1).
- (vi) Interest rate R satisfies FOC of equation (2).
- (vii) Capital rental price r satisfies the non-arbitrage condition $r - \delta = R$. Capital market clearing condition requires $K = \int k_i di$.
- (viii) Labor market is cleared: $N = \int n_i di$.
- (ix) Household budget constraint equation (3) is satisfied.

5. Inflation, Production Efficiency and Markup Distortion

5.1 Efficient Outcome

In this section, we discuss the main focus of this paper: the production efficiency loss under various long-run inflation levels.

First, we examine the production frontier that final output production is facing. Conditional on the total resource of N and K , there is a production possibility frontier that governs the trade-off between intermediate goods. Consider the intermediate goods production function $y_i = a n_i (k_i/n_i)^{1-\gamma}$. Since capital and labor input markets are perfectly competitive, each intermediate producers will share the same capital-to-labor input ratio. Therefore, the intermediate production frontier of $\{y_i\}$ can be expressed as a hyperplane of $\int y_i di = a N^\gamma K^{1-\gamma}$.⁷ Express the hyperplane in terms of vintages:

⁷ $\int y_i di = a (k/n)^{1-\gamma} \int n_i di = a (k/n)^{1-\gamma} N$. Furthermore, $K/N = \int k_i di / \int n_i di = [(k/n) \int n_i di] / \int n_i di = k/n$.

$$\sum_{k=0}^{\infty} \alpha(1-\alpha)^k y_k = aN^\gamma K^{1-\gamma}. \quad (13)$$

An optimal final output production must maximize the final output production function:

$$y = \left[\sum_{k=0}^{\infty} \alpha(1-\alpha)^k y_k^\rho \right]^{\frac{1}{\rho}}, \quad (14)$$

subject to the production frontier equation (13).

Look at the vintage bundle $y_v \equiv \sum_{k=1}^{\infty} \alpha(1-\alpha)^{k-1} y_k^\rho$. It can be shown that the optimization problem of maximizing equation (14) subject to a linear constraint like $\sum_{k=0}^{\infty} \alpha(1-\alpha)^k g_k y_k = C$ will yield an optimal vintage bundle that is equivalent to maximizing the following:⁸

$$y = \left[\alpha y_0^\rho + (1-\alpha) y_v^\rho \right]^{\frac{1}{\rho}}, \quad (15)$$

subject to

$$\alpha g_0 y_0 + (1-\alpha) g_v y_v = C,$$

where $g_v = \left[\sum_{k=1}^{\infty} \alpha(1-\alpha)^{k-1} g_k^{-\rho/(1-\rho)} \right]^{-(1-\rho)/\rho}$ serves as an optimal price index for y_v bundle.

Apply the above argument to the production frontier equation (13); it implies that $g_v = 1$. We can phrase the optimal choice of y_0 and y_v as maximizing equation (15) subject to:

$$\alpha y_0 + (1-\alpha) y_v = aN^\gamma K^{1-\gamma}. \quad (16)$$

Objective function equation (15) suggests a marginal rate of technical substitution (*MRTS*) between y_0 and y_v as:

$$MRTS = \frac{\alpha y_0^{\rho-1}}{(1-\alpha) y_v^{\rho-1}}.$$

⁸ See Appendix 2.

The production frontier equation (16) constraint suggests a marginal rate of transformation (*MRT*) between y_0 and y_v as:

$$MRT = \frac{\alpha}{(1 - \alpha)}.$$

The first order condition for efficient production, which requires $MRTS = MRS$, is:

$$MRTS \equiv \frac{\alpha y_0^{\rho-1}}{(1 - \alpha) y_v^{\rho-1}} = MRT \equiv \frac{\alpha}{1 - \alpha}. \tag{17}$$

It implies that $y_0 = y_v = y$. Evenly employed intermediate inputs are the key for intermediate input usage efficiency.

There is a second layer of efficiency concerning labor input usage. Under intermediate input usage efficiency, $y = aN^\gamma K^{1-\gamma}$. An efficient labor input level must maximize $u(c, N)$. This requires that the marginal rate of substitution (*MRS*) between consumption and labor to be equal to the marginal productivity of labor:

$$MRS \equiv -\frac{u_N}{u_c} = MPN \equiv \gamma a \left(\frac{K}{N}\right)^{1-\gamma}. \tag{18}$$

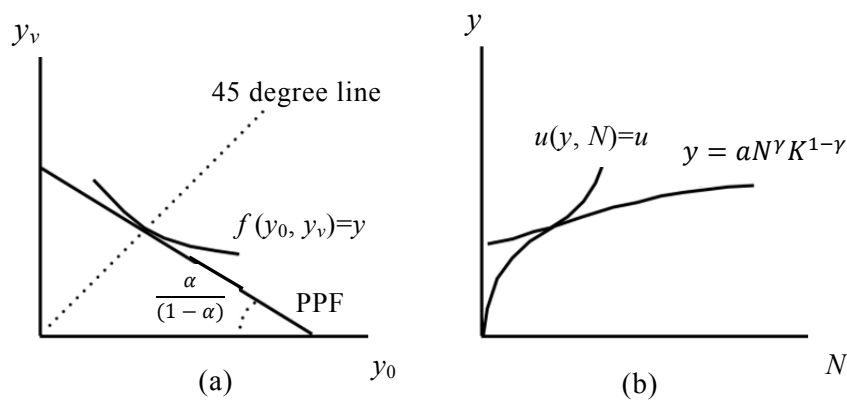


Figure 1 Efficient Outcomes

Figure 1 panel (a) shows the efficient intermediate input usage condition (17) and panel (b) shows the efficient labor input condition (18).

5.2 Two Channels of Distortion

In earlier section, we highlight two efficiency conditions in the economy. One is the intermediate input usage efficiency from $MRTS = MRT$; the other is the labor input efficiency from $MRS = MPN$. In this section, we will discuss how inflation can distort these two efficiency conditions through two channels. One is called the price variation channel; the other is called the marginal markup channel.

For price variation, we refer to the relative price between the zero vintage goods and the vintage bundle, which is q_0/q_v where:

$$q_v = \left[\sum_{k=1}^{\infty} \alpha(1-\alpha)^{k-1} q_k^{\frac{-\rho}{(1-\rho)}} \right]^{\frac{-(1-\rho)}{\rho}}, \quad (19)$$

with $q_k = P_k/P$. The final goods producer minimizes the cost $\sum_{k=0}^{\infty} \alpha(1-\alpha)^k q_k y_k$ subject to $y = (\sum_{k=0}^{\infty} \alpha(1-\alpha)^k y_k^{\rho})^{1/\rho}$. Its dual problem is maximizing $(\sum_{k=0}^{\infty} \alpha(1-\alpha)^k y_k^{\rho})^{1/\rho}$ subject to $\sum_{k=0}^{\infty} \alpha(1-\alpha)^k q_k y_k = C$ for some constant C . As we argue earlier in Section 5.1, its optimal vintage bundle choice of y_v is the same as maximizing $y = (\alpha y_0^{\rho} + (1-\alpha)y_v^{\rho})^{1/\rho}$ subject to $\alpha q_0 y_0 + (1-\alpha)q_v y_v = C$ so long as q_v is defined as equation (19). As a result, in competitive equilibrium, $MRTS$ will be equal to the input price ratio:

$$MRTS = \frac{\alpha q_0}{(1-\alpha)q_v}.$$

When $q_v/q_0 \neq 1$, $MRTS$ will be different from $MRT = \alpha/(1-\alpha)$. When there is a positive inflation rate, $q_0/q_v > 1$ and

$$MRTS = \frac{\alpha}{1-\alpha} \times \frac{q_0}{q_v} > MRT = \frac{\alpha}{1-\alpha}. \quad (20)$$

Note that $q_k = q_0 \pi^{-k}$. It follows that:⁹

$$\frac{q_v}{q_0} = g(\pi) = \left[\sum_{k=1}^{\infty} \alpha (1-\alpha)^k \pi^{\frac{(k+1)\rho}{(1-\rho)}} \right]^{-\frac{(1-\rho)}{\rho}} = \frac{1}{q_0 \pi}. \quad (21)$$

As we saw earlier $\partial q_0 / \partial \pi > 0$, it follows that $g'(\pi) < 0$. The higher the inflation the cheaper the vintage bundle y_v relative to the newly-priced goods y_0 . Under the reasonable parameter range, $g'(\pi) < 0$; a higher inflation makes the vintage-price goods cheaper than the newly-priced good. Holding others equal, we learn that higher inflation rate will enhance the discrepancy in equation (20) and induce a larger intermediate input usage distortion.

Through the price variation channel, inflation aggravates not only the intermediate input usage inefficiency, but also the labor input usage inefficiency. To see the latter point, consider the aggregate markup $\mathcal{M} \equiv P/\Psi$ where P is the aggregate price defined by equation (11) and Ψ is the nominal marginal cost of intermediate goods production. We can rewrite the aggregate markup as a geometric average between marginal markup $m_0 = P_0/\Psi = q_0/\psi$ and the vintage markup $m_v = P_v/\Psi = q_v/\psi$ as:¹⁰

$$\mathcal{M} = \left[\alpha m_0^{\frac{-\rho}{(1-\rho)}} + (1-\alpha) m_v^{\frac{-\rho}{(1-\rho)}} \right]^{\frac{-(1-\rho)}{\rho}},$$

⁹ Note that $\left[\sum_{k=0}^{\infty} \alpha (1-\alpha)^k \pi^{(k+1)\rho/(1-\rho)} \right]^{-(1-\rho)/\rho} = \left[\alpha / 1 - (1-\alpha) \pi^{\rho/(1-\rho)} \right]^{-(1-\rho)/\rho} \pi^{-1} = q_0^{-1} \pi^{-1}$ using equation (21).

¹⁰ $\mathcal{M} = \left\{ \left[\sum_{k=0}^{\infty} \alpha (1-\alpha)^k \rho_k^{-\rho/(1-\rho)} \right]^{-(1-\rho)/\rho} \right\} / \Psi = \left[(\alpha (\rho_0/\Psi)^{-\rho/(1-\rho)} + (1-\alpha) (\rho_v/\Psi)^{-\rho/(1-\rho)}) \right]^{-(1-\rho)/\rho}$.

which is the average markup as defined in King and Wolman (1996). Note that $m_v/m_0 = q_v/q_0$. Therefore,

$$\mathcal{M} = m_0 \left[\alpha + (1 - \alpha) \left(\frac{q_v}{q_0} \right)^{\frac{-\rho}{(1-\rho)}} \right]^{\frac{-(1-\rho)}{\rho}}. \quad (22)$$

The average markup is affected by the marginal markup m_0 and the price variation q_v/q_0 .

As inflation increases q_v/q_0 , working through the price variation channel it also increases the average markup. However, we also see another channel that inflation can affect the average markup, which is the marginal markup channel. As we show in the appendix that the higher the inflation the higher the marginal markup, i.e. $m_0'(\pi) > 0$. Through marginal markup channel, inflation enhances the average markup further.

Note that even with zero inflation ($\pi = 1$), the average markup $\mathcal{M} = m_0 = 1/\rho > 1$. Hence, the higher the inflation, the further away the average markup is from one. Focus on labor input efficiency and consider the final output $y = aN^\gamma K^{1-\gamma}$. With the constant-return-to-scale technology, the real marginal cost of production for each intermediate producer can be expressed as w/MPN .¹¹ It means that the real marginal cost $\psi = w/MPN$. Since $1/\psi = \mathcal{M}$, with proper rearrangement, we obtain:

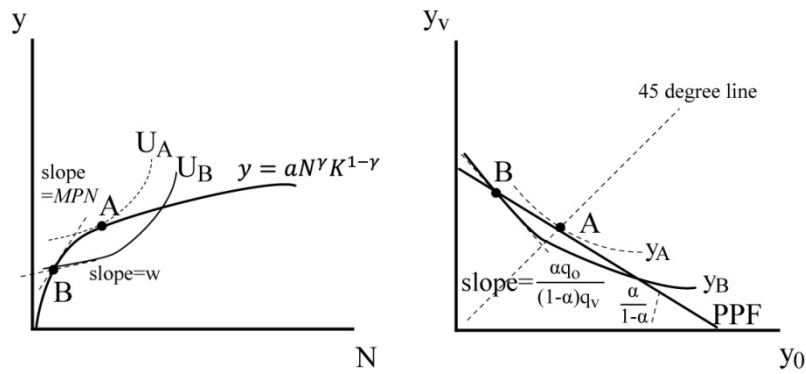
$$w = \frac{MPN}{\mathcal{M}} < MPN, \quad \mathcal{M} > 1.$$

Therefore, in an environment with $\mathcal{M} > 1$,

$$MRS \equiv \frac{-u_N}{u_C} = w < MPN.$$

¹¹ With CRTS, real marginal cost is equal to real average cost so that $\psi = (wN + \gamma K)/y$ and $MPN \times N + MPK \times K = y$. Therefore, we have $\psi = w[(1/MPN) - (MPK/MPN)(K/y)] + r(K/y) = w/MPN$. The last equality using the FOC for cost minimization $MPK/MPN = r/w$.

The final good output level will be too low as is the employment level. As $w = MPN/\mathcal{M}$, higher inflation increases the average markup (through both the price variation channel and the marginal markup channel) and further aggravates the labor usage inefficiency.



(a) Shows Markup Distortion (b) Shows Price Variation Distortion

Figure 2 Distortion Under Inflation

Figure 2 panel (a) shows the outcome difference between the efficient outcome, point A, and the markup distorted outcome, point B. The discrepancy between wage and MPN sometimes is treated as a distortionary tax on the wage income as in Galí (2009). Figure 2 panel (b) shows the outcome difference between the efficient outcome, point A, and the price-variation distorted outcome, point B. The final output level associated with B, y_B , is lower than the one associated with A, y_A . Note that both points are in the same PPF which means they use the same labor input. However, point B means that the society uses the same labor input to generate less final output when there is price variation.

5.3 Endogenize the Price Adjustment Speed

In a higher inflation environment, intermediate producers will

charge a higher marginal markup m_0 . This generates an effect on increasing the average markup distortion size. In addition to that, the relative price variation is larger, i.e. a smaller q_v/q_0 , which makes vintage-price intermediate goods more appealing for final production. However, due to convex production technology of final goods, this means that less final goods can be produced. Positive long-run inflation rate only enhances inefficiency. Evidently, there is no justification for targeting a positive long-run inflation rate.

Nonetheless, the above argument is based on a fixed price adjustment speed α . Intuitively, firms have an incentive to increase their price adjustment speed when facing a higher inflation situation. Adjusting price more frequently can reduce the magnitude of price erosion that a higher inflation creates. Under this situation, a society facing higher inflation does not necessarily encounter a greater distortion through both the marginal markup and the price variation channels.

To endogenize the adjustment speed, we introduce a constant adjustment cost A in terms of the final output loss. This is like a menu cost as Mankiw (1985). For the convenience of discussion, we denote the model as Calvo-AC model. We focus on long-run equilibrium. The choice of α is treated as a social norm that each firm chooses at the very beginning of their existence. A soon-to-be intermediate producer knows that in the long run it has $\alpha(1-\alpha)^k$ chance to be in vintage k and receives the value v_k . It means that α must maximize the following steady state expected value:

$$\alpha(v_0 - A) + \sum_{k=1}^{\infty} \alpha(1-\alpha)^k v_k.$$

We show in the appendix that the above objective function is equivalent to:

$$\frac{1}{1-\beta} \sum_{k=1}^{\infty} \alpha(1-\alpha)^k \phi(q_k) - \frac{1}{1-\beta} \alpha A. \quad (23)$$

The second expression has a clear economic meaning. For the term $\sum_{k=0}^{\infty} \alpha(1-\alpha)^k \phi(q_k)$, it represents the long-run expected profit that a firm can get from a given sticky price path. And αA is the expected adjustment cost to pay in the steady state. The expected profit net of adjustment cost in the steady state is $\sum_{k=0}^{\infty} \alpha(1-\alpha)^k \phi(q_k) - \alpha A$.

We can drop $1-\beta$ from objective function (23) since it does not affect the choice of α . Furthermore, with $\phi(q) = (q-\psi)(q)^{-1/(1-\rho)}$ and $q_k = q_0 \pi^{-k}$, the objective function can be expressed as:

$$\begin{aligned} & V(\alpha, \pi, q_0, A, \psi, y) \\ &= \left[\frac{1}{1-(1-\alpha)\pi^{\frac{\rho}{1-\rho}}} q_0^{\frac{-\rho}{1-\rho}} - \frac{1}{1-(1-\alpha)\pi^{\frac{1}{1-\rho}}} q_0^{\frac{-1}{1-\rho}} \psi \right] y - \alpha A. \end{aligned}$$

Under zero inflation rate ($\pi = 1$), the value function is equal to $\left[q_0^{-\rho/(1-\rho)} - q_0^{-1/(1-\rho)} \psi \right] y - \alpha A$. Furthermore, $q_0 = \psi/\rho$, following the static markup rule as implied by equation (10). In this case, the optimal adjustment speed is not to adjust at all, i.e. $\alpha = 0$, since adjusting price only incurs adjustment cost without any benefit gain.

From the optimal pricing condition, we can express the initial pricing $q_0 = q(\alpha, \pi, \psi)$. Then we can express the α choice objective as $V[\alpha, \pi, q(\alpha, \pi, \psi), A, \psi, y]$. The FOC for optimal α choice is :

$$\frac{\partial V[\alpha, \pi, q(\alpha, \pi, \psi), A, \psi, y]}{\partial \alpha} + \frac{\partial V[\alpha, \pi, q(\alpha, \pi, \psi), A, \psi, y]}{\partial q} \frac{\partial q}{\partial \alpha} = 0. \quad (24)$$

Given a steady state inflation level π , a steady state equilibrium for Calvo-AC is a choice of α and an allocation $[c, N, K, y, (y_k, n_k, k_k)_{k=0}^{\infty}]$ with the price mechanism $[(q_k)_{k=0}^{\infty}, r, w, R]$ that satisfy the conditions in Section 4 together with equation (24).

Senay and Sutherland (2014) applied a similar endogenous flexibility setting as ours. The only difference is the cost of adjustment is modeled as a form of labor input waste. Their paper focuses on the optimal choice of monetary policy under zero long-run inflation rate. There are other endogenous price flexibility studies on the long-run horizon, such as Devereux (2006) who assumes heterogeneous adjustment cost and uses a two-stage information structure in a static model to discuss the long-run exchange rate policy issue.

6. Model Calibration

We proceed to analyze the long-run effect of long-run inflation with model calibration. First we explain the choice of model parameter values. The capital depreciation rate δ is set to 0.0135 which is based on the long-run average of the depreciation capital cost to capital stock value ratios from the annual data from 1929 to 2006.¹² The following parameters are set in a standard way as in Dotsey et al. (1999): The utility discounting factor $\beta=0.984$ which implies a real return of 6.5 percent annually; The elasticity of intermediate input demand is set to 4.33 which implies a markup of 1.3 in a flexible pricing economy; labor supply is infinitely elastic ($\sigma = 0$), which can be interpreted as the result of optimal labor contracts in the presence of indivisible labor as in Hansen (1985) and Rogerson (1988). The total productivity level a is normalized to 1.

As to the preference parameter ξ and the production parameter γ , they are tuned to target a labor's share of two-thirds and the working time that accounts for 20 percent of the life time. The resulting values

¹² Data is from BEA and the capital includes all fixed assets and consumer durable goods. The average depreciation rate is 0.054 per year.

of these two parameters depend on the price adjustment frequency and the inflation level. The benchmark inflation rate is 1 percent quarterly.¹³

Starting from a standard Calvo model with a fixed adjustment probability, we set $\alpha = 0.25$. It implies an expected price contract length of 4 quarters for each intermediate producers. With an inflation rate of one percent, the targets of labor share and the working time, it indicates that $\xi = 0.2858$ and $\gamma = 0.8648$. Table 1 summarizes all the values.

Table 1 calibrates the model, we choose an adjustment cost level that can justify $\alpha = 0.25$ as the optimal choice under the benchmark quarterly inflation rate of one percent. This sets $A = 0.006$. The total adjustment cost to output ratio (i.e. $\alpha A/\gamma$) in the benchmark point is only 0.65 percent. This shows that a rather small adjustment cost is capable of generating significant price stickiness in which the expected contract length is one year.

Table 1 Parameter Values

Household	
β	0.9840
σ	0.0000
ξ	0.2858
δ	0.0135
Final goods producers	
ρ	0.7690
Intermediate goods producers	
a	1.0000
γ	0.8648
Inflation and price stickiness	
π	1.0100
α	0.2500
Adjustment cost	
A	0.0060

Source: Based on author's own setting.

¹³ Without further notification, all inflation rates are quarterly rate.

6.1 Standard Calvo Model

This paper focuses on the relationship between long-run inflation and the production efficiency trade-off. Two important wedges are highlighted: the average markup \mathcal{M} and the relative price q_0/q_v .

Figure 3 shows the relationship between inflation rate and two wedges. As we can see, the higher the inflation rate, the larger the relative price and the larger the average markup. This means that inflation serves no good but increases the distortions in the economy.

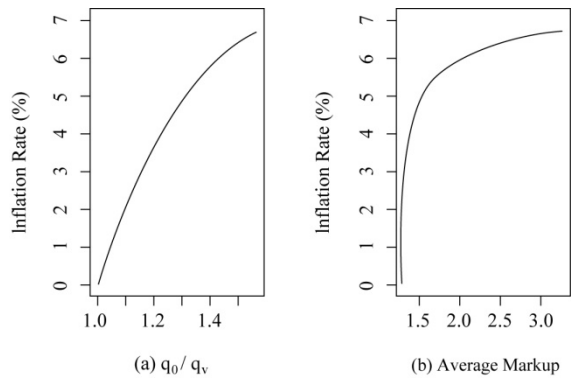


Figure 3 Relative Price and the Average Markup under Calvo

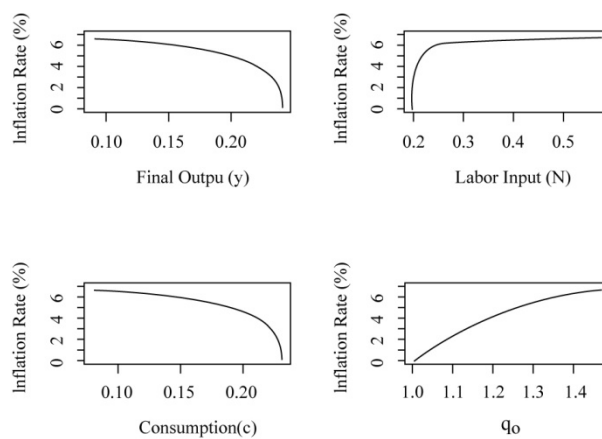


Figure 4 The Economy under Calvo

Figure 4 shows how the distortions affect the labor employment (N), final output (y), consumption (c) and q_0 . As distortions get large with inflation (as seen in Figure 3), final output decreases. Labor employment increases. This is an inefficiency phenomenon: more labor input but less output.

The output contractionary effect of inflation in the long run under time-dependent contracts was also discovered in several papers. Graham and Snower (2004) focused on the Taylor type staggered wage contract. They found that long-run inflation is output contractionary except in the mild inflation regime. Casares (2004) also studied the Calvo pricing model, focusing on the effect of markup distortion (that is the capital tax channel in his paper). The same long-run contractionary result is obtained. He also found that this result is robust across several versions of the pricing rule with exogenous price adjustment frequency.

6.2 Calvo Model with Adjustment Cost

Now we move to Calvo-AC. Figure 5 panel (a) shows that a higher inflation rate is associated with a faster price adjustment. The intuition behind this is the effect of α on mitigating inflation erosion. Consider the real profit function $\phi(q) = (q - \psi)q^{-1/(1-\rho)}y$ which reaches its maximal under the static markup $1/\rho$. When $q/\psi > 1/\rho$, a higher q reduces the profit; when $q/\psi < 1/\rho$, a lower q decreases the profit. We know that marginal markup $m_0 = q_0/\psi$ is increasing with inflation as it is firm's incentive to insure against inflation erosion and it is larger than $1/\rho$. Therefore, a higher inflation will decrease zero vintage profit (i.e. $\phi(q_0)$ will decrease).¹⁴ Furthermore, $q_k = q_0\pi^{-k}$ which is discounted by π at an exponential rate. It means that a higher inflation will always dominate the direction of q_k change in large k . Therefore, a higher inflation, though increasing q_0 , will decrease q_k eventually for k large enough. Since $q_k/\psi < 1/\rho$ for large k , a further decrease

¹⁴ Actually, $\phi(q_k)$ will decrease for k small enough.

in q_k implies $\phi(q_k)$ decreases further. Overall the steady state expected value of an intermediate producer $\sum_{k=0}^{\infty} \alpha (1 - \alpha)^k \phi(q_k)$ is likely to decrease. In such a case, choosing a higher price adjustment speed helps the firm to insure against such a negative change. It does not only decrease the chance of receiving lower $\phi(q_k)$ from the future, but also give an incentive to set a lower marginal markup as we discussed about the optimal pricing equation (10), which mitigates the negative change of $\phi(q_0)$ as well.¹⁵

Figure 5 panel (b) indicates that price variation is still increasing with inflation, which is similar to the implication from the standard Calvo model. Higher inflation makes firms to price higher whenever they could—even though they tend to adjust their prices more often. The big difference is in panel (c). Given that $\rho = 0.7690$, the static markup is 1.3004 ($1/\rho$). As we can see, the average markup can go below the static markup level. This is different from the standard Calvo model where static markup is the lowest average markup an economy can reach.

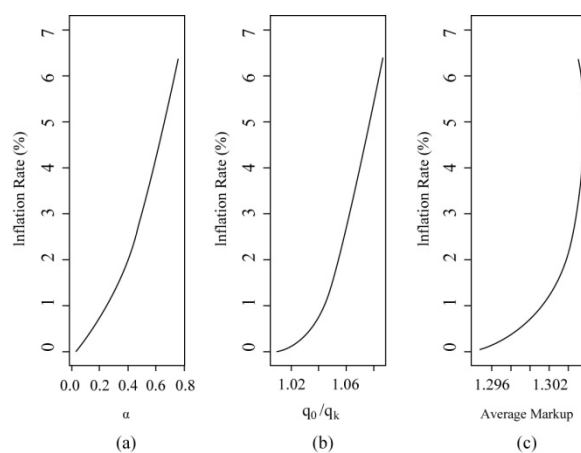
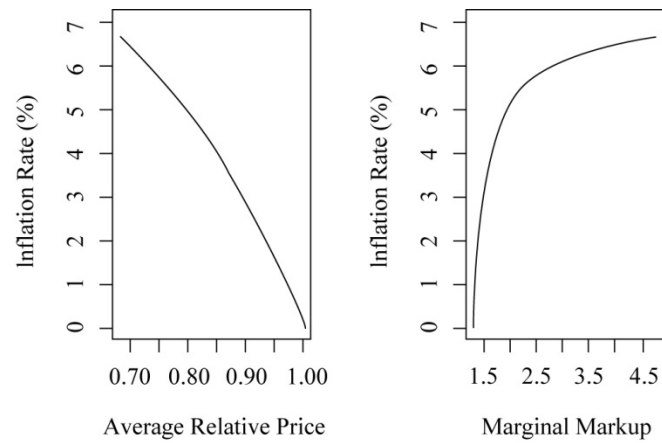


Figure 5 Price Adjustment Speed, Relative Price and the Average Markup under Calvo-AC

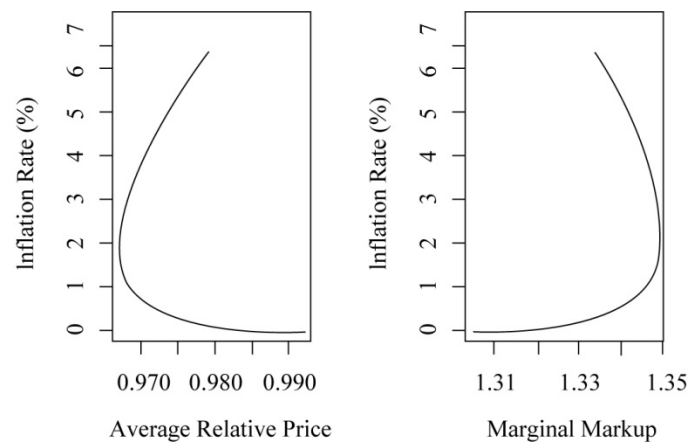
¹⁵ A rigorous proof is to look at equation (24). However, we fail to give a clear analytical result. But intuitively, when the adjustment cost is small as in our simulation, the narrative we gave should dominate.

To understand the reason that the average markup can go below the static markup, we need to see its two composites: the marginal markup m_0 and the average relative price $[\alpha + (1 - \alpha)(q_v/q_0)^{-\rho/(1-\rho)}]^{-(1-\rho)/\rho}$ as in equation (22). Figure 6 compares the difference of the average relative price and the marginal markup m_0 between Calvo and Calvo-AC models. Before two percent inflation rate, both models have the same qualitative pattern. The higher the inflation rate, the higher the average relative price and the marginal markup. However, in Calvo-AC the average relative price is much closer to one, and the marginal markup m_0 is also very close to the static markup (1.30). The latter creates a room for the economy to bring down the average markup below the static one. The reason that marginal markup is not so sensitive to inflation is as follows. When inflation is low the price adjustment speed is also low. The population of cheap vintage-priced goods is larger. Setting a high m_0 means that the firms will have a hard time to compete with more low markup firms in the equilibrium. This keeps m_0 from being too high. When inflation rate is high, the price adjustment speed is high. The incentive to charge a high markup to mitigate price erosion effect diminishes. This also keeps m_0 from being too high. As a result, though marginal markup can still be affected by inflation as in standard Calvo, the range of its variation is much confined.

Another difference that Figure 6 shows is that the relationship among average relative price, marginal markup and inflation can reverse when the inflation rate is high. Similarly in Figure 5 panel (c), the average markup trade-off can curve back in the high inflation region. However, it does not mean that a high inflation target can be efficiency improving. The two distortions we consider do not take into account the loss of adjustment cost. When inflation rate is high, the adjustment speed is faster; the entire economy bears a larger adjustment cost.



(a) Standard Calvo Model



(b) Calvo-AC Model

Figure 6 Average Relative Price and Marginal Markup: A Comparison

Consider the result that both average markup and the relative price decreases as inflation rate drops. An efficient outcome seems to be an outcome under zero inflation rate as the standard Calvo model has suggested. However, our calibrated result actually does support a minor inflation for efficiency. In Figure 5 panel (c), average markup actually

risers back to the static markup level when the inflation rate is zero (the detailed discussion of which will be in Section 6.4). Therefore, a minor inflation rate is justifiable for mitigating the average markup distortion.

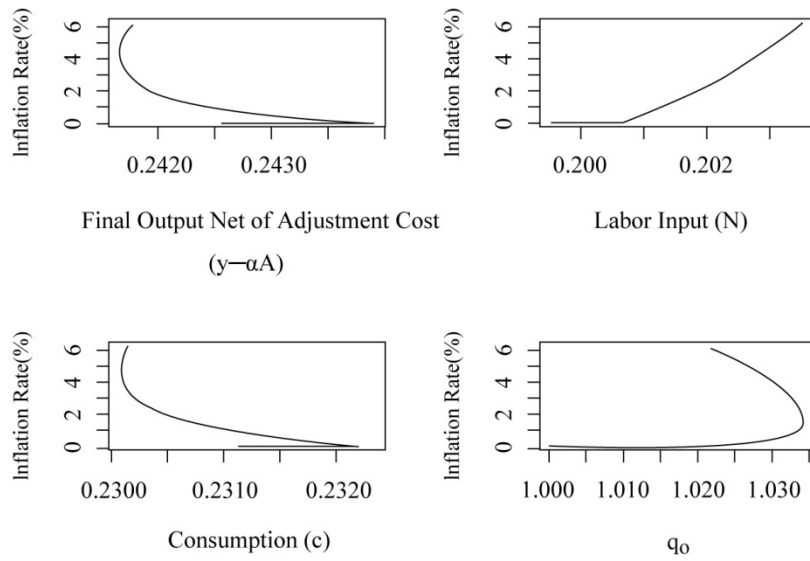


Figure 7 The Economy under Calvo-AC

We turn to the economic activities as shown in Figure 7. We focus on the region with an inflation rate that is above 0.01 percent. For close to zero inflation region, we will zoom in the figure and discuss later. Figure 7 shows that a higher inflation has a negative impact on the net final output and consumption. The price variation distortion increases with inflation which induces a larger labor demand. Vintage zero price q_0 increases with inflation so as to insure against price erosion. However, when inflation is high enough, the adjustment speed is high enough to mitigate the worry of price erosion. This shows in the pattern that q_0 trade-off curve back in high inflation region. In high inflation region, we also see net final output and consumption increase with inflation. This is consistent with efficiency improvement from the

average markup's decreasing with inflation, as we see in Figure 5 panel (c). However, the increase of net final output and consumption has a limit when it reaches the point that $\alpha = 1$ (fully flexible pricing). It can not be better than the zero inflation rate case in the long-run since high inflation bears a high adjustment cost but zero inflation does not (because $\alpha = 0$ in such case).

6.3 Efficiency under Minor Inflation

Through U.S. history from 1947:Q1 to 2008:Q2, quarterly inflation rate never exceeds 5 percent.¹⁶ The highest level was 3.66 percent in 1951:Q1. Below 5 percent range, the calibrated result in Calvo-AC looks qualitatively similar to standard Calvo. Inflation is contractionary in final output and consumption. However, for Calvo-AC if we zoom in the minor inflation region, minor inflation actually is expansionary.

We can see that minor inflation actually increases both output (as well as net-of-AC output) and consumption.¹⁷ The reason is because minor inflation brings the average markup below the static $1/\rho$ markup which corrects the markup distortion to some degree (see column (c) in Figure 8). In Calvo-AC, price adjustment frequency is very low in low inflation environment. As a result, there will be more old vintage prices on the left tail of the price distribution. This will pull down the average markup.

¹⁶ Based on GDP deflator.

¹⁷ From zero to 0.1 percent in Calvo-AC.

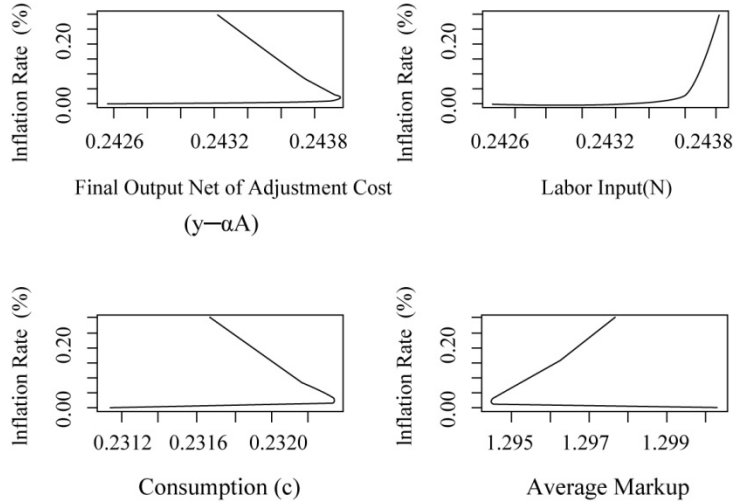


Figure 8 Minor Inflation under Calvo-AC

6.4 Production Efficiency and Production Function Convexity

We learn that under Calvo-AC minor inflation can be better than zero inflation. In this section, we compute the range of inflation that is considered minor inflation. We first compute the consumption (c_0), net final output (y_0^N) and the utility (u_0) under zero inflation rate. Then look for the inflation rate upper bound so that: when the inflation rate is below that bound, consumption, net final output and the utility will be larger than the zero inflation rate situation. The upper bound for three different criteria: consumption larger than c_0 , net output larger than y_0^N , utility higher than u_0 are written in the first three rows, “ $> c_0$ ”, “ $> y_0^N$ ” and “ $> u_0$ ” respectively, in Table 2. We also compute the range under different ρ values other than 0.769 as we did in the earlier calibration. The main reason that we explore different ρ values is because: the strength of two inflation distortions (average markup and price variation) are closely related to the convexity property of the final goods production. As ρ (between zero and one) approaches one, the

production function approaches linearity and the substitutability between intermediate goods is stronger; the uneven intermediate input usage produces less production inefficiency. Furthermore, higher substitutability brings down markup.

Starting from $\rho = 0.769$, our pervious calibration setting, inflation rates lower than 0.1 percent quarterly will generate a steady state equilibrium that has consumption, net final output and utility higher than the zero inflation outcomes. Any positive inflation rate below 0.1 percent is minor enough to improve efficiency and utility level from the zero inflation situation. When ρ is smaller (like 0.3, 0.4), the upper bound also increases. For $\rho = 0.4$, any inflation rate below 0.7 percent will generate an equilibrium consumption and output that are larger than the zero inflation outcomes. The wider range is because, in this case, intermediate producers have a stronger monopoly power that creates a bigger average markup distortion to be mitigated. A higher inflation is capable of generating a faster price adjustment speed that may bring down the average markup as we discussed earlier. A stronger monopoly power gives a wider range of inflation to mitigate the average markup distortion. When ρ gets bigger, the monopoly power is weaker and the minor inflation range shrinks.

We also calculate the inflation target that can generate maximal utility ($\max u$), maximal consumption ($\max c$) and maximal net final output ($\max y^N$). For the benchmark $\rho = 0.769$, 0.01 percent quarterly inflation rate can generate the maximal utility, 0.02 percent quarterly rate can generate maximal consumption and net final output. The inflation targets get larger when firms' monopoly power is stronger (i.e. ρ gets smaller).

The last row in the table shows the inflation bound that can generate the average markup less than the static markup. This bound also gets larger when the monopoly power gets stronger.

Table 2 Efficiency, Welfare and Inflation

Calvo-AC		ρ				
		0.3	0.4	0.769	0.8	0.9
Upper bound for:	$c > c_0$	0.0090	0.0070	0.0010	0.0010	0.0003
	$y^N > y_0^N$	0.0090	0.0070	0.0010	0.0010	0.0003
	$u > u_0$	0.0120	0.0080	0.0010	0.0007	0.0001
		Targets				
The required inflation rate to hit the targets	$\max u$	0.0006	0.0004	0.0001	0.0001	0.0000
	$\max c$	0.0007	0.0006	0.0002	0.0002	0.0001
	$\max y^N$	0.0007	0.0006	0.0002	0.0002	0.0001
Upper bound for:	average markup $< 1/\rho$	0.0220	0.0190	0.0070	0.0060	0.0030

Source: Based on author's own setting.

This paper is limited to long-run analysis. Future extension to embody short-run dynamics would be an interesting extension. However, as Ascari (2004) points out, along the non-zero inflation rate path the forward-looking Phillips curve can no longer be condensed to finite future expectation terms. Therefore, short-run dynamic analysis is challenging, including the local stability examination. Though we did not conduct local stability analysis in this paper, the inflation rates we experiment are far below 7.5 percent in Ascari's numerical analysis, under which the dynamic system is stable.

7. Conclusion

Inflation affects output in the long run through two channels. They are the average markup channel and the price variation channel. Average markup larger than one means the output level is too low to be efficient. Price variation of the intermediate inputs implies that the given final output level is not produced in an efficient way. These two distortions give long-run inflation the wedge to maneuver the real output in the long run.

We study standard Calvo models with exogenous and endogenous price adjustment frequency. For the endogenous version, we introduce a fixed adjustment cost and let firms choose their long run adjustment frequency. We find that price adjustment speed increases with inflation.

For standard Calvo with exogenous price adjustment speed, higher inflation induces a larger price variation as well as a higher average markup. There is no efficiency gain from targeting a positive inflation level. However, when we move to the endogenous adjustment frequency models, we find that in the low inflation environment the economic outcome might be better than in the zero inflation environment. This is because the price adjustment frequency is low in the low inflation regime. With price erosion from inflation, this means that there will be more producers charging a markup lower than the zero inflation markup. Average markup can go below the static markup level (which prevails under zero inflation). In this case, output level inefficiency can be partially corrected. This efficiency gain can overcome the efficiency loss from price variation that is induced by inflation. Our result justifies a mild positive long-run inflation target.

We also find that the optimal long-run inflation target under the endogenous adjustment frequency models should be lower if the market competition is stronger. This is because the justification to targeting a positive inflation comes from average markup distortion correction. When the market is more competitive, the resulting markup is already small. The justification becomes weak. To prevent the inefficiency of price variation from outweighing the average markup distortion correction, the optimal inflation target has to be kept lower.

Appendix 1 Effect of Inflation on Marginal Markup

For notation simplicity, let $\tilde{\beta} = \beta(1 - \alpha)$. We consider the case where the static equilibrium exists, which requires:

$$\tilde{\beta}\pi^{\frac{1}{1-\rho}} < 1, \quad (\text{A1})$$

$$\tilde{\beta}\pi^{\frac{\rho}{1-\rho}} < 1. \quad (\text{A2})$$

The equilibrium marginal markup can be expressed as:

$$\frac{q_0}{\psi} = \frac{1}{\rho} \times \frac{1 - \tilde{\beta}\pi^{\frac{\rho}{1-\rho}}}{1 - \tilde{\beta}\pi^{\frac{1}{1-\rho}}}.$$

Taking the first derivative of $\rho q_0/\psi$ with respect to π , we obtain

$$\frac{1}{\Delta^2} \times \frac{1}{(1-\rho)} \left[-\rho \tilde{\beta} \pi^{\frac{(2\rho-1)}{(1-\rho)}} \left(1 - \tilde{\beta} \pi^{\frac{1}{1-\rho}} \right) + \tilde{\beta} \pi^{\frac{\rho}{1-\rho}} \left(1 - \tilde{\beta} \pi^{\frac{\rho}{1-\rho}} \right) \right],$$

where $\Delta = (1 - \tilde{\beta}\pi^{1/(1-\rho)})$. With proper rearrangement, the term inside the bracket [.] is equal to:

$$(1 - \rho) \tilde{\beta} \pi^{\frac{2\rho}{1-\rho}} \left[\pi^{\frac{-1}{1-\rho}} - \tilde{\beta} \right] + \tilde{\beta} \pi^{\frac{\rho}{1-\rho}} \left[1 - \frac{1}{\pi} \right].$$

First, with positive inflation rates, $1 - \pi^{-1} > 0$. Secondly, the condition (A1) implies $\pi^{-1/(1-\rho)} - \tilde{\beta} > 0$. Together they imply that the first derivative of the marginal markup to inflation is positive. The higher the inflation, the larger the marginal markup.

Appendix 2 Condense Model to y_0 and y_v

The final output function is

$$y = \left[\sum_{k=0}^{\infty} \alpha (1 - \alpha)^k y_k^\rho \right]^{\frac{1}{\rho}}. \quad (\text{A3})$$

Consider maximizing (A3) with respect to $\{y_k\}_{k=0}^{\infty}$ subject the following type of linear constraint.

$$\sum_{k=0}^{\infty} \alpha (1 - \alpha)^k g_k y_k = C.$$

Let $y_v = (\sum_{k=1}^{\infty} \alpha (1 - \alpha)^{k-1} y_k^\rho)^{1/\rho}$ denote the vintage bundle. Then $y = (\alpha y_0^\rho + (1 - \alpha) y_v^\rho)^{1/\rho}$. Based on Separation Theorem, we can conduct a two-step maximization to obtain the equivalent optimal choice.

First, we can solve the vintage bundle choice problem:

$$y_v = \max_{\{y_k\}} \left[\sum_{k=1}^{\infty} \alpha (1 - \alpha)^{k-1} y_k^\rho \right]^{\frac{1}{\rho}}, \quad (\text{A4})$$

$$\text{s.t. } \sum_{k=1}^{\infty} \alpha (1 - \alpha)^{k-1} g_k y_{-k} = C_v.$$

To obtain $y_v = y_v(C_v, \{g_k\}_{k=1}^{\infty})$. Then solve:

$$\max_{\{y_0, C_v\}} \{ \alpha y_0^\rho + (1 - \alpha) y_v [C_v, \{g_k\}_{k=1}^{\infty}]^\rho \},$$

$$\text{s.t. } \alpha g_0 y_0 + (1 - \alpha) C_v = C. \quad (\text{A5})$$

For problem (A4), the FOCs are

$$\frac{y_k}{y_1} = \left(\frac{g_k}{g_1}\right)^{\frac{-1}{(1-\rho)}} \text{ for all } k = 1, 2, \dots, \infty.$$

$$C_v = \sum_{k=1}^{\infty} \alpha (1-\alpha)^{k-1} g_k y_k.$$

Therefore,

$$\begin{aligned} C_v &= \sum_{k=1}^{\infty} \alpha (1-\alpha)^{k-1} g_k \left(\frac{g_k}{g_1}\right)^{\frac{-1}{(1-\rho)}} y_1 \\ &= \sum_{k=1}^{\infty} \alpha (1-\alpha)^{k-1} (g_k)^{\frac{-\rho}{(1-\rho)}} g_1^{\frac{-1}{(1-\rho)}} y_1. \end{aligned}$$

Hence

$$g_1^{\frac{1}{(1-\rho)}} y_1 = C_v \left[\sum_{k=1}^{\infty} \alpha (1-\alpha)^{k-1} (g_k)^{\frac{-\rho}{(1-\rho)}} \right]^{-1}. \quad (\text{A6})$$

And

$$\begin{aligned} y_v &= \left[\sum_{k=1}^{\infty} \alpha (1-\alpha)^{k-1} \left(\frac{g_k}{g_1}\right)^{\frac{-\rho}{(1-\rho)}} y_1^\rho \right]^{\frac{1}{\rho}} \\ &= \left[\sum_{k=1}^{\infty} \alpha (1-\alpha)^{k-1} g_k^{\frac{-\rho}{(1-\rho)}} \right]^{\frac{1}{\rho}} g_1^{\frac{1}{(1-\rho)}} y_1 \\ &= \left[\sum_{k=1}^{\infty} \alpha (1-\alpha)^{k-1} (g_k)^{\frac{-\rho}{(1-\rho)}} \right]^{\frac{(1-\rho)}{\rho}} \times C_v. \end{aligned}$$

The last equality follows from (A6). It follows that

$$C_v = \left[\sum_{k=1}^{\infty} \alpha (1-\alpha)^{k-1} (g_k)^{\frac{-\rho}{(1-\rho)}} \right]^{\frac{-(1-\rho)}{\rho}} y_v.$$

Let $g_v \equiv \left[\sum_{k=1}^{\infty} \alpha (1-\alpha)^{k-1} g_k^{-\rho/(1-\rho)} \right]^{-(1-\rho)/\rho}$. Then problem (A5) is equal to

$$\begin{aligned} \max_{\{y_0, C_v\}} & \left[\alpha y_0^\rho + (1-\alpha) \left(\frac{C_v}{g_v} \right)^\rho \right] \\ \text{s.t.} & \alpha g_0 y_0 + (1-\alpha) C_v = C. \end{aligned} \tag{A7}$$

which is the same as

$$\begin{aligned} \max_{\{y_0, y_v\}} & \left[\alpha y_0^\rho + (1-\alpha) y_v^\rho \right] \\ \text{s.t.} & \alpha g_0 y_0 + (1-\alpha) g_v y_v = C. \end{aligned} \tag{A8}$$

and let $C_v = g_v y_v$ once we solve for y_v .

Appendix 3 The Objective Function for Choosing α

In the steady state,

$$v_0 = \phi(q_0) + \beta\alpha(v_0 - A) + \beta(1 - \alpha)v_1,$$

$$v_k = \phi(q_k) + \beta\alpha(v_0 - A) + \beta(1 - \alpha)v_{k+1}.$$

The second equation implies that

$$\begin{aligned} \alpha \sum_{k=1}^{\infty} (1 - \alpha)^k v_k &= \sum_{k=1}^{\infty} \alpha(1 - \alpha)^k \phi(q_k) + \beta\alpha(1 - \alpha)(v_0 - A) \\ &\quad + \beta\alpha \sum_{k=1}^{\infty} (1 - \alpha)^k v_k - \beta\alpha(1 - \alpha)v_1. \end{aligned}$$

Therefore,

$$\begin{aligned} \alpha \sum_{k=1}^{\infty} (1 - \alpha)^k v_k &= \frac{1}{(1 - \beta)} \sum_{k=1}^{\infty} \alpha(1 - \alpha)^k \phi(q_k) \\ &\quad + \frac{\beta\alpha(1 - \alpha)}{(1 - \beta)}(v_0 - A) - \frac{\beta\alpha(1 - \alpha)}{(1 - \beta)}v_1. \end{aligned}$$

It follows that

$$\begin{aligned} &\alpha(v_0 - A) + \alpha \sum_{k=1}^{\infty} (1 - \alpha)^k v_k \\ &= \frac{1}{(1 - \beta)} \sum_{k=1}^{\infty} \alpha(1 - \alpha)^k \phi(q_k) + \frac{\alpha(1 - \alpha\beta)}{(1 - \beta)}(v_0 - A) - \frac{\alpha\beta(1 - \alpha)}{(1 - \beta)} \\ &= \frac{\alpha(1 - \alpha)}{(1 - \beta)} \sum_{k=0}^{\infty} \alpha(1 - \alpha)^k \phi(q_{k+1}) + \frac{\alpha(1 - \alpha\beta)}{(1 - \beta)}(v_0 - A) - \frac{\alpha\beta(1 - \alpha)}{(1 - \beta)}. \quad (\text{A9}) \end{aligned}$$

In the steady state,

$$v_0 - A = [\phi(q_0) - A] + \beta\alpha(v_0 - A) + \beta(1 - \alpha)v_1.$$

Therefore,

$$(1 - \alpha\beta)(v_0 - A) = \phi(q_0) - A + \beta(1 - \alpha)v_1. \quad (\text{A10})$$

Putting (A10) into (A9), we obtain that

$$\begin{aligned} & \alpha(v - A) + \alpha \sum_{k=1}^{\infty} (1 - \alpha)^k v_k \\ &= \frac{\alpha}{(1 - \beta)} \sum_{k=0}^{\infty} (1 - \alpha)^k \phi(q_k) - \frac{\alpha A}{(1 - \beta)}. \end{aligned} \quad (\text{A11})$$

Appendix 4 The Optimal Choice of α

We can drop $1 - \beta$ from objective function (A11). Furthermore, with $\phi(q) = (q - \psi)(q)^{-1/(1-\rho)}y$,

$$\sum_{k=0}^{\infty} (1 - \alpha)^k \phi(q_k) = \sum_{k=0}^{\infty} (1 - \alpha)^k q_k^{\frac{-\rho}{(1-\rho)}} - \sum_{k=0}^{\infty} (1 - \alpha)^k q_k^{\frac{-1}{(1-\rho)}} \psi,$$

where q is the real price charged, ψ is the real marginal cost and y is the aggregate final output. Since $q_k = q_0 \pi^{-k}$, the objective function can be expressed as:

$$\begin{aligned} & V(\alpha, \pi, q_0, A, \psi, y) \\ &= \alpha \left[\frac{1}{1 - (1 - \alpha)\pi^{\frac{\rho}{(1-\rho)}}} q_0^{\frac{-\rho}{(1-\rho)}} - \frac{1}{1 - (1 - \alpha)\pi^{\frac{1}{(1-\rho)}}} q_0^{\frac{-1}{(1-\rho)}} \psi \right] y - \alpha A. \end{aligned}$$

From the optimal pricing condition (10), we can express the initial pricing $q_0 = q(\alpha, \pi, \psi)$, ignoring the deep parameters β and ρ for a concise reason. Then we can express the α choice objective as $V[\alpha, \pi, q(\alpha, \pi, \psi), A, \psi, y]$. The FOC for optimal α choice is

$$\frac{\partial V[\alpha, \pi, q(\alpha, \pi, \psi), A, \psi, y]}{\partial \alpha} + \frac{\partial V[\alpha, \pi, q(\alpha, \pi, \psi), A, \psi, y]}{\partial q} \frac{\partial q}{\partial \alpha} = 0.$$

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Calvo 價格僵固模型下長期物價上漲率的影響

林茂廷

摘 要

長期物價上漲率在 Calvo 價格僵固模型下會產生兩種生產無效率，一是來自於平均加碼扭曲 (average markup distortion)，另一是來自於價格差異扭曲 (price variation distortion)。前者指的是平均加碼高於邊際成本，造成總產出過少的無效率；後者指的是異質性中間財的價格差異造成中間要素僱用不平均的要素使用無效率。在典型的 Calvo 模型裡，價格的調整速度為外生，此時越高的長期物價上漲率只會強化兩種生產無效率，因此看不出維持長期物價上漲的任何好處。然而，當我們引入調整成本，內生化價格調整速度，微幅的長期物價上漲雖然會強化中間財的價格差異，卻可以降低平均加碼，而後者帶來的效率提升是有可能超過前著的效率損傷。本研究亦指出，若中間要素市場越競爭，則最適長期物價上漲率目標要越低。

關鍵詞：Calvo 價格僵固模型、物價上漲率長期目標

JEL 分類代號：E31, E50, E52

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投稿日期：民國 101 年 9 月 24 日；修訂日期：民國 102 年 4 月 8 日；
接受日期：民國 103 年 3 月 31 日。