

Spreads, Depths, and Optimal Submission Strategies of Market and Limit Orders

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Abstract

This paper analyzes a monopolistic market maker's joint determination of quoted spreads and depths, as well as informed and uninformed traders' optimal submission strategies under various trading environments. The influence of quoted depths on traders' order-submission decisions is found to be significant. Therefore, the market maker can adjust quoted depths to avoid unfavorable trades on one side or both sides of the market. In addition, it is shown that traders are more willing to submit market orders when spreads decrease, depths increase, same-side limit order books are thicker, or expected future market order flows decrease. Conversely, limit orders are preferred.

Keywords: Bid-ask Spreads, Quoted Depths, Market Orders, Limit Orders

JEL Classification: G12, G14, C72

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1. Introduction

In addition to bid-ask spreads, the importance of quoted depths in measuring market liquidity and as market-maker's strategic instrument against adverse selection has been well recognized by both theoretical (Kavajecz, 1998; Chen et al., 2000; Dupont, 2000) and empirical (Lee et al., 1993; Kavajecz, 1999; Kavajecz and Odders-White, 2001) studies. Nevertheless, almost all the theoretical models simultaneously endogenizing both spreads and depths make an unrealistic assumption, i.e., asset traders can submit market orders only. Domowitz (1993) and Harris and Hasbrouck (1996) point out that limit orders are conspicuous trading instruments.¹ Biais et al. (1995) find evidence supporting rising market liquidity with more limit orders submitted. Glosten (1994) also concludes that the exclusion of limit orders from traders' choices may cause biased inferences about market liquidity under the setup of electronic open limit order books. The main objective of this paper is, thus, to investigate how a market maker determines his equilibrium spreads and depths jointly when his trading opponents could choose between market and limit orders.

A static quote-driven trading model with informed and uninformed traders, and a monopolistic market maker is constructed. All players are strategic.² The market maker sets a spread and depth first, then either an informed or uninformed trader submits orders. These orders are

¹ According to Harris and Hasbrouck (1996), approximately 45 percent of the total orders submitted by investors using the Super Dot system at the New York Stock Exchange (NYSE) are limit orders.

² This means that both informed and uninformed traders maximize their expected utility given market maker's quotations. On the other hand, the market maker chooses his quotations to maximize the expected profit given traders' optimal submission strategies.

executed against the limit order book or at the market maker's quotations. Unlike the models of Kavajecz (1998), Chen et al. (2000) and Dupont (2000), both market and limit orders are in traders' choices here. Moreover, our uninformed traders are presumed risk averse instead of risk neutral such as in Kavajecz's (1998) and Dupont's (2000) setups. In this study, how limit order books and market maker's quotations affect traders' optimal submission strategies is first investigated. Then, the impact of trading-environment parameters on market maker's equilibrium quotations is examined through numerical analyses. The parameters include: degree of adverse selection, variance of private information, risk-averse level of uninformed traders, magnitude of liquidity shocks, size of expected future market order flows, and thickness of limit order books.

The conditions for informed and uninformed traders to optimally submit limit or market orders are derived. It is discovered that market maker's quotations, especially quoted depths, have significant influence on traders' optimal submission strategies. When quoted depths increase, traders are likely to submit market orders due to higher execution probability. This in turn implies that the market maker can quote small depths to induce informed traders to submit limit orders. This is consistent with Kavajecz's (1999) empirical finding, which shows that specialists can manage their quoted depths to reduce the adverse selection risk. In contrast, when quoted spreads increase, traders are more likely to submit limit orders due to higher transaction costs. Thick limit order books would lower the probability of traders' submission of same-side limit orders, but raise the submission chance of opposite-side market orders. The reason is pretty intuitive. Since when more unexecuted limit orders are accumulated, the newly submitted same-side limit orders are less likely to be executed. But thick limit

order books would provide enough depths for the execution of opposite-side market orders, thus would encourage their submission. In contrast, Parlour (1998) uncovers that thicker one-side limit order books will result in lower submission rates of the-other-side market orders.³ At last, when traders expect large future market order flows, they would prefer limit to market orders for higher expectation of achieving profitable transactions.

Our numerical solutions show that, under specific condition, the market maker could avoid one-side transactions totally by quoting small depths. This is consistent with Kavajecz's (1999) empirical finding. The required condition is that the one-side limit order books of the market must be thinner and large enough future market order flows of the other-side are expected. When the condition fails, the market maker would provide similar large depths on both sides of the market. Moreover, it is observed that the market maker would raise spreads and reduce depths when informed traders are more likely present in the market. As variances of private information rise, both equilibrium spreads and depths would increase. This is consistent with Liu et al. (2001) empirical finding. Same-direction movements of equilibrium spreads and depths also occur when uninformed traders become more risk averse and liquidity shocks are larger in magnitude. Nevertheless, when limit order books are thicker, the market maker would raise spreads and reduce depths to protect himself if he desires to trade on both sides of the market; and he would reduce both spreads and depths if one-side trading is intended only. Finally, the market maker would reduce spreads and raise depths as future market order flows are

³ Detailed explanations and the comparisons between Parlour's and our models are provided after Corollary 1.

expected to rise.

If limit orders set prices within the quoted bid-ask spread, then they are aggressive limit orders. This situation is investigated in our extension section, and their execution probabilities are assumed endogenous. Under the situation, prior behavior outcomes for informed and uninformed traders remain true qualitatively. And the market maker may still conduct one-side trading. Moreover, it is found that when more aggressive limit orders are submitted, the market maker is more likely to do one-side transactions.

Static models considering limit order books include studies of Kyle (1985), Easley and O'Hara (1987), Angel (1990), Glosten (1994), Kumar and Seppi (1994), Chakravarty and Holden (1995), Rock (1996), Handa and Schwartz (1996), Seppi (1997), and Harris (1998). Kyle (1985) and Easley and O'Hara (1987) explore competitive order-driven trading systems. In their setups, schedules of quoted prices at all quantities are offered by competitive market makers. This is different from our design where a monopolistic market maker quotes merely one pair of spread and depth at each time period. Easley and O'Hara (1987) focus on how investors' exogenous trading amounts affect market makers' spreads and security prices. In contrast, by allowing investors to choose between limit and market orders and endogenizing quoted depths, we inspect how market maker's quotations affect investors' optimal submission strategies. While Rock (1996) treats limit orders as requests for specific quantities of a security at specific prices, Angel (1990) and Harris (1998) presume that traders submit fixed-unit limit orders and behaviors of market makers and uninformed traders are exogenous.

On the other hand, Glosten (1994) endogenizes trading quantities and considers no market makers. Kumar and Seppi (1994), Seppi (1997),

and Chakravarty and Holden (1995) allow some but not all of the players to be strategic. In these setups, traders can choose between market and limit orders and market maker's quoted depths are set exogenous. In particular, Chakravarty and Holden (1995) develop a quote-driven trading model, as we do, and show that informed traders may submit limit orders with prices within the range of bid-ask spreads. Assuming both informed and uninformed traders simultaneously submitting orders under specific circumstance, it is optimal for them to submit market-buys and limit-sells (or market-sells and limit-buys) orders at the same time, instead of placing market orders only. In addition to the presumption of sequential order submission, our setup differs from Chakravarty and Holden's (1995) in letting uninformed traders be strategic and considering endogenous quoted depths. Chakravarty and Holden (1995) suppress the role of limit order books, while our model can analyze the impact of limit order books on traders' order submission strategies.

Moreover, Wald and Horrigan (2005), Kaniel and Liu (2006), and Foucault et al. (2007) all allow investors to optimally choose the submission of market or limit orders. However, the trading quantities are exogenous even prices set by the orders are endogenous in their models. Beber and Caglio (2005) provide the evidence that informed traders will submit limit orders for the purpose of hiding information. And Anand et al. (2005) empirically show that informed traders are more likely to adopt market orders at the beginning of and limit orders at the end of a trading day, while uninformed traders behave oppositely.

With respect to dynamic setups in the literature, studies like Parlour (1998), Foucault (1999), Hollifield et al. (1999), Goettler et al. (2005), and Foucault et al. (2005) allow investors to submit both market and limit orders at various degrees of aggressiveness and endogenize

the execution probability of limit orders. Nevertheless, none of the studies inspects the influence of players' asymmetric information on their own behaviors.

The rest of this paper is organized as follows. Our model is presented in Section 2. Then, informed and uninformed traders' optimal submission strategies are derived in Sections 3 and 4, respectively. Monopolistic market maker's optimization problems are discussed in Section 5. In Section 6, simulation results are reported and analyzed. The robustness of our outcomes is shown in Section 7. In Section 8, we summarize the differences between our findings and those of previous works. Finally, conclusions are drawn in Section 9.

2. Model

There are three types of strategic players in our model: (i) risk-neutral informed traders, (ii) risk-averse uninformed traders, and (iii) a monopolistic risk-neutral market maker.⁴ A risky asset is traded. We focus on one trading period, t , in a market day. The proposed static quote-driven trading mechanism proceeds as follows. Prior to period t , the market maker posts his bid and ask prices, and corresponding depths. The unexecuted limit orders accumulated up to time $(t-1)$ are also posted to all traders. Then either an informed or uninformed trader arrives and submits market or limit orders. Thereafter, orders are

⁴ The settings of investors' behaviors in previous studies dealing with quoted depths are somewhat ad hoc. For instance, Kavajecz (1998) assumes that an uninformed trader will buy (sell) when the ask (bid) price is below (above) his reservation level. Thus, we try to provide some micro-foundations by assuming that uninformed traders are endowed with negative exponential utility functions and try to maximize their expected utility. However, for simplicity, informed traders are still presumed to be risk neutral. Suppose they are risk averse, the outcomes in Sections 3-5 will remain true qualitatively. But the comparative statics results displayed in Section 6 may change.

executed at market maker's quotations or against unexecuted limit orders, and the value of the risky asset is realized. As in Parlour (1998), it is assumed that each trader arrives only once during a market day. And suppose that submitted limit orders cannot be canceled, but would expire at the end of the market day.⁵ On the other hand, to emphasize the immediate execution nature of market orders, all unfilled market orders at period t are assumed automatical expiration as entering period $(t+1)$ For investors demanding trading amounts greater than quoted depths plus amounts contained in limit order books, they can only take quantities currently available in the market because investors can transact merely once during the market day.⁶

When the market clears at time t , the value of the risky asset, v , is realized, where v is given by

$$v = \bar{V} + \eta$$

with \bar{V} being a known constant and $\eta \sim N[0, \sigma_\eta^2]$. And η is observable to informed traders only. With probability $\alpha \in (0, 1)$, an informed trader will be present at time t . In contrast, with probability $(1 - \alpha)$, an uninformed trader encountering an endowment (or liquidity) shock would like to trade the risky asset to rebalance his portfolio position. Although the monopolistic market maker does not know the exact type of the traders present, he is aware of the value of α .

Denote by p^a and p^b the market maker's quoted ask and bid prices, respectively. And q^a and q^b equal the corresponding quoted

⁵ According to Harris and Hasbrouck's (1996), 82-percent limit orders in the Super Dot system are day orders.

⁶ Our model can be thought to deal with the small-order case. This case is interesting because of its prevalence in the marketplace. Please see Handa and Schwartz (1996), Harris (1998), and Wald and Horrigan (2005) for detailed discussions of this issue.

depths. Let $B > 0$ and $S > 0$ be the unexecuted limit-buys and limit-sells accumulated up to period $(t-1)$ at the bid and ask prices, respectively. (B, S) is then the limit order book at time t . Both informed and uninformed traders can submit market and limit orders. As in Parlour (1998), if a market-sell (market-buy) order is submitted, it will be transacted at p^b (p^a). And if a limit-sell (limit-buy) order is submitted at t , it will be transacted at p^a (p^b). Since only one trader is allowed at each time period, limit orders submitted at t would be executed afterwards and the fulfilled quantity depends on coming market orders. The assumption of all limit orders being placed at the same price makes orders in limit order books have execution priority over newly submitted limit orders and market maker's quotations in matching with arriving market orders. Thus, traders submitting market orders suffer price disadvantage, but have execution priority. Unlike in Hollifield et al. (1999), our model does not consider the possibility that limit orders will be picked off, which is so-called the winner's curse. Hence, traders submitting limit orders enjoy price gains, but bear the uncertainty about the realized trading quantities.

Let random variables \tilde{M}^B and \tilde{M}^S be the amounts of market-buys and market-sells arriving after time t , respectively. \tilde{M}^B and \tilde{M}^S are assumed to distribute over corresponding intervals $[\underline{M}^B, \bar{M}^B]$ and $[\underline{M}^S, \bar{M}^S]$, where $0 < \underline{M}^B < \bar{M}^B < \infty$ and $0 < \underline{M}^S < \bar{M}^S < \infty$. The cumulative density functions of \tilde{M}^B and \tilde{M}^S are respectively denoted by $G^B(\cdot)$ and $G^S(\cdot)$ which are common knowledge to all traders. Accordingly, we define the random variable of future execution rate of a b^l -unit limit-buy submitted at t , \tilde{F}^{b^l} , as

$$\tilde{F}^{b^l} \equiv \begin{cases} 0 & \text{if } \tilde{M}^s \leq B, \\ \min\{1, \frac{\tilde{M}^s - B}{b^l}\} & \text{if } \tilde{M}^s > B. \end{cases} \quad (1)$$

Similarly, the random variable of future execution rate of a s^l -unit limit-sell submitted at t is

$$\tilde{F}^{s^l} \equiv \begin{cases} 0 & \text{if } \tilde{M}^B \leq S, \\ \min\{1, \frac{\tilde{M}^B - S}{s^l}\} & \text{if } \tilde{M}^B > S. \end{cases} \quad (2)$$

Based on \tilde{F}^{b^l} and \tilde{F}^{s^l} , traders can determine the order type they will submit.

Moreover, no borrowing and lending constraints are considered here. Thus, transaction prices deviating from \bar{V} are caused by adverse selection when the market maker trades with informed traders. That is,

$$p^a = \bar{V} + \gamma, \quad (3)$$

$$p^b = \bar{V} - \gamma, \quad (4)$$

where γ is one half of the bid-ask spread with $0 < \underline{\gamma} \leq \gamma \leq \bar{\gamma} < \infty$. And q^a may not equal to q^b .

Finally, there are two assumptions implicit in the above model. The first is that traders are not allowed to submit aggressive limit orders. That is, we consider limit order books merely at market maker's quotations. The second is that the execution probability of limit orders is exogenous. That is, the distributions of \tilde{M}^B and \tilde{M}^S are independent of the prices and quantities set by submitted orders and limit order books, as well as the time left before the end of a market day. These assumptions will be relaxed later to check whether findings would change.

3. Informed Traders' Optimal Strategies

In this section, informed traders' optimal trading quantities using market and limit orders for any given η are first derived. Then, by comparing profits acquired under the two order types, informed traders' optimal submission strategies are known.

Define b_i^m and b_i^l to be the quantities of market-buys and limit-buys submitted by informed traders, respectively. And s_i^m and s_i^l label respectively the quantities of market-sells and limit-sells submitted. If an informed trader places a b_i^m -unit market-buy, given η , γ and q^a , then his expected profit is

$$E\pi^l(b_i^m | \eta, \gamma, q^a) = b_i^m(v - p^a) = b_i^m(\eta - \gamma), \quad (5)$$

with $b_i^m \leq (q^a + S)$. Since the market-buy will be traded at p^a , the maximum transaction amount at time t is $(q^a + S)$. If an informed trader places a b_i^l -unit limit-buy under the same given conditions, his expected profit is

$$E\pi^l(b_i^l | \eta, \gamma, q^a) = b_i^l E(\tilde{F}^{b_i^l})(v - p^b) = b_i^l E(\tilde{F}^{b_i^l})(\eta + \gamma), \quad (6)$$

with $b_i^l E(\tilde{F}^{b_i^l}) \leq \bar{S} - B$. Here $b_i^l E(\tilde{F}^{b_i^l})$ is the expected units the informed trader can transact during the remaining market day by submitting limit-buys now, and \bar{S} is the market-sell amount expected to arrive during the remaining market day.⁷ Since limit order books

⁷ That is, $E(\tilde{F}^{b_i^l}) = \int_B^{\bar{M}^s} \min\{1, (\bar{M}^s - B)/b_i^l\} \cdot dG^s(\bar{M}^s) / [G^s(\bar{M}^s) - G^s(B)]$ and $\bar{S} = E(\bar{M}^s) \equiv \int_{\underline{M}^s}^{\bar{M}^s} \bar{M} dG^s(\bar{M}^s)$. And $E(\tilde{F}^{s_i^l})$ and \bar{B} are notations applied to the opposite situation. One can replace \bar{S} and \bar{B} by other statistics, such as median. Our results will remain true. However, how informed traders expect future market-order flows matters in a dynamic model, which characterizes interactions of different trading periods.

have execution priority, $\bar{S} - B$ is the maximum trading quantities achievable by a limit-buy at time t .

Similarly, given η , γ and q^b , if an informed trader submits an s_i^m -unit market-sell and an s_i^l -unit limit-sell with $s_i^l, s_i^m > 0$, then his expected profits are

$$E\pi^l(s_i^m | \eta, \gamma, q^b) = -s_i^m(\eta + \gamma), \quad (7)$$

with $s_i^m \leq (q^b + B)$, and

$$E\pi^l(s_i^l | \eta, \gamma, q^b) = s_i^l E(\tilde{F}^{s_i^l})(-\eta + \gamma), \quad (8)$$

with $s_i^l E(\tilde{F}^{s_i^l}) \leq (\bar{B} - S)$, respectively. Here \bar{B} is the market-buy amount expected to arrive during the remaining market day, and $s_i^l E(\tilde{F}^{s_i^l})$ is the expected unit the informed trader can transact during the remaining market day by submitting limit-sells now.

For any η , we can derive the optimal order quantities when market-buys, limit-buys, market-sells, or limit-sells are submitted. In sum, informed traders would buy (sell) as many as possible when private information is good (bad) enough. For instance, when a limit-buy is submitted, the optimal amount, b_i^{l*} , must solve the problem of

$$\begin{aligned} \max_{b_i} & \quad b_i^l E(\tilde{F}^{b_i^l})(\eta + \gamma) \\ \text{s.t.} & \quad b_i^l E(\tilde{F}^{b_i^l}) \leq (\bar{S} - B). \end{aligned}$$

By applying Lagrangian method, we get $b_i^{l*} E(\tilde{F}^{b_i^*}) = (\bar{S} - B)$ when $\eta > -\gamma$.⁸ Similarly, it is optimal for informed traders to buy $(q^a + S)$ units when a market-buy is submitted and $\eta > \gamma$. Selling $(q^b + B)$ units is the best for informed traders when market-sells are submitted and $\eta < -\gamma$. And placing an s_i^{l*} -unit limit-sell with $s_i^{l*} E(\tilde{F}^{s_i^*}) = (\bar{B} - S)$ is optimal for informed traders when $\eta < \gamma$.

Thereafter, by comparing expected profits generated at various optimal order quantities, we can find informed traders' optimal submission strategies, which are summarized in Lemma 1.

Lemma 1. *Suppose $(\bar{B} - S) > 0$, $(\bar{S} - B) > 0$, $\eta^{**} \equiv -\gamma(\bar{S} - B + q^a + S) / (\bar{S} - B - q^a - S)$ and $\eta^{*} \equiv -\gamma(q^b + B + \bar{B} - S) / (q^b + B - \bar{B} + S)$ with $\eta^{b*} \geq \gamma > 0 > -\gamma \geq \eta^{s*}$. Then we have the followings.*

- (i) *If the expected maximum trading amount achievable by submitting market orders is lower than that by placing limit orders, informed traders will submit limit orders only. That is, when $(\bar{S} - B) > (q^a + S)$ and $(\bar{B} - S) > (q^b + B)$, then informed traders would submit limit-buys for all $\eta \in (0, \infty)$, and limit-sells for all $\eta \in (-\infty, 0)$.*

⁸ We cannot obtain the exact value of b_i^{l*} without specifying the function form of $G^b(\cdot)$. Fortunately, what really matters in our model is the value of $b_i^{l*} E(\tilde{F}^{b_i^*})$ which is shown to equal $\bar{S} - B$. Accordingly, the specification of $G^b(\cdot)$ becomes irrelevant. Similar arguments can be applied to s_i^{l*} . In the following, an example is provided to show how to derive b_i^{l*} and s_i^{l*} . Suppose \tilde{M}^B is uniformly distributed over the interval $[\underline{M}^B, \bar{M}^B]$. Then $G^b(\tilde{M}^B) = \tilde{M}^B - \underline{M}^B / \bar{M}^B - \underline{M}^B$ and $b_i^{l*} = [2(\bar{M}^B - B) + \sqrt{(\bar{M}^B - B)^2 - 8(\bar{S} - B)(\bar{M}^B - \underline{M}^B)}] / 2 > 0$ for a sufficiently large \bar{M}^B . Similarly, if \tilde{M}^S is uniformly distributed over the interval $[\underline{M}^S, \bar{M}^S]$. Then $s_i^{l*} = [2(\bar{M}^B - S) + \sqrt{(\bar{M}^B - S)^2 - 8(\bar{B} - S)(\bar{M}^B - \underline{M}^B)}] / 2 > 0$ for a large enough \bar{M}^B .

- (ii) For cases excluded from (i), informed traders' order submission depends on the value of η
- (iia) If $(\bar{S} - B) > (q^a + S)$ and $(\bar{B} - S) < (q^b + B)$, then informed traders would submit limit-buys as $\eta \in (-\gamma, \infty)$, market-sells as $\eta \in (-\infty, \eta^{s*}]$, and limit-sells as $\eta \in [\eta^{s*}, \gamma)$.
- (iib) If $(\bar{S} - B) < (q^a + S)$ and $(\bar{B} - S) > (q^b + B)$, then informed traders would submit market-buys as $\eta \in [\eta^{b*}, \infty)$, limit-buys as $\eta \in (-\gamma, \eta^{b*}]$, and limit-sells as $\eta \in (-\infty, \gamma)$.
- (iic) If $(\bar{S} - B) < (q^a + S)$ and $(\bar{B} - S) < (q^b + B)$, then informed traders would submit market-buys as $\eta \in [\eta^{b*}, \infty)$, limit-buys as $\eta \in (-\gamma, \eta^{b*}]$, limit-sells as $\eta \in [\eta^{s*}, \gamma)$, and market-sells as $\eta \in (-\infty, \eta^{s*}]$.

Proof. See the Appendix.

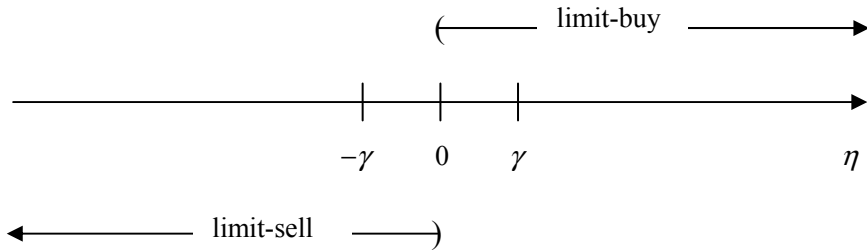
The outcomes of Lemma 1(i), 1(iia), 1(iib), and 1(iic) are drawn in Graphs 1-4, respectively. Lemma 1 demonstrates that informed traders' optimal submission strategies are determined by relative magnitudes of expected maximum trading volumes achievable using limit and market orders, and the values of η . The intuition on the bid-side of the market is provided below. Given expected maximum trading volumes achievable using limit-buys and market-buys, $\bar{S} - B$ and $q^a + S$, respectively, an informed trader with $\eta > 0$ would obtain

$$(\eta + \gamma)(\bar{S} - B), \quad (9)$$

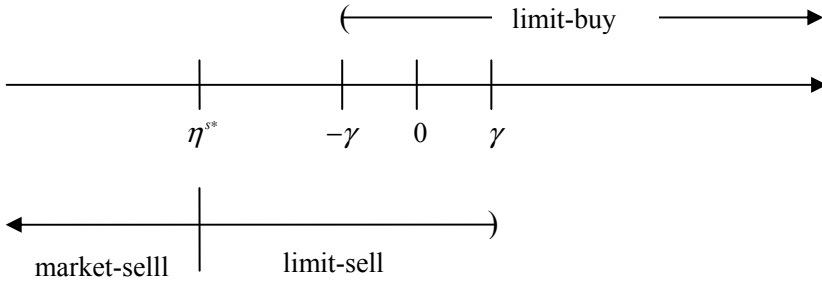
as limit-buys are submitted, and

$$(\eta - \gamma)(q^a + S), \quad (10)$$

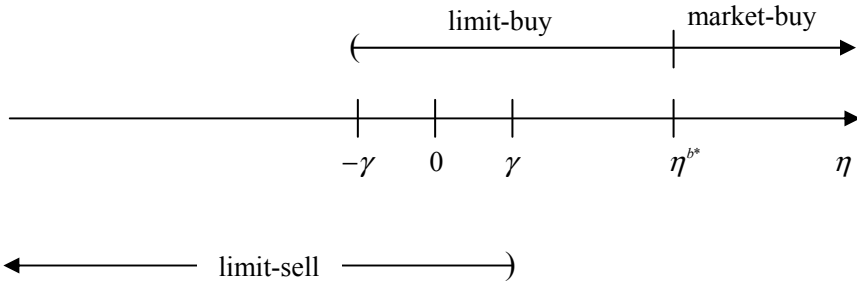
as market-buys are placed. Since $\eta + \gamma > \eta - \gamma$, limit-order users own the unit-price advantage. Thus, when $\bar{S} - B > q^a + S$, limit-buys are the better choice for the informed trader whatever η 's value is. This is exactly Lemma 1(i) claims. However, the result may alter when $q^a + S > \bar{S} - B$. Under the latter case, the informed trader's optimal submission strategies would depend on how good the private information he owns. If the information is not very good (i.e., $\eta < \eta^{b*}$), limit-buys are still the informed trader's best choice. Oppositely, if the information is good enough (i.e., $\eta \geq \eta^{b*}$), the effect of large trading amount would outweigh unit-price disadvantage for market orders. Accordingly, the informed trader will submit market-buys. By the same arguments, the informed trader will submit market-sells when $q^b + B > \bar{B} - S$ and the private information is bad enough (i.e., $\eta \leq \eta^{s*}$). Otherwise, limit-sells are the better choice. These are the contents of Lemma 1(ii).



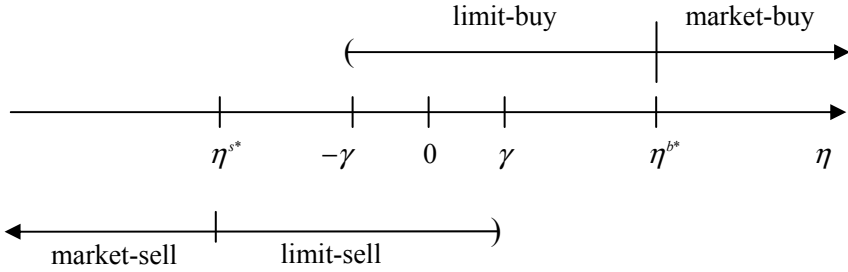
Graph 1 $\bar{S} - B > q^a + S$ and $\bar{B} - S > q^b + B$



Graph 2 $\bar{S} - B > q^a + S$ and $\bar{B} - S < q^b + B$



Graph 3 $\bar{S} - B < q^a + S$ and $\bar{B} - S > q^b + B$



Graph 4 $\bar{S} - B < q^a + S$ and $\bar{B} - S < q^b + B$

It is worth mentioning that the market maker can avoid transacting with informed traders with high $|\eta|$ by managing his quotations. For instance, the market maker can set larger γ to raise η^{b*} and $|\eta^{s*}|$, then limit orders are more likely submitted by informed traders. If the market maker believes that $\eta > \eta^{b*}$ is very likely, he can quote a sufficiently small q^a (i.e., $q^a < \bar{S} - B - S$) to induce informed traders' submission of limit-buys. This result is consistent with Kavajecz's (1999) finding, which shows that specialists manage their quoted depths to reduce the adverse selection risk. In addition, limit order books and expected future market order flows could affect the probability of market order submission through η^{b*} and η^{s*} .

Finally, Lemma 1 implies that the smaller are η^{b*} and $|\eta^{s*}|$, the more likely will market orders be submitted. Corollary 1 thus summarizes that how market maker's quotations, limit order books, and expected market order flows affect informed traders' submission strategies.

Corollary 1. *Suppose $(q^a + S) > (\bar{S} - B) > 0$ and $(q^b + B) > (\bar{B} - S) > 0$. Then we have the followings.*

- (i) *With greater quoted spreads, informed traders are more likely to submit limit orders. That is,*

$$\partial \eta^{b*} / \partial \gamma = -(\bar{S} - B + q^a + S) / (\bar{S} - B - q^a - S) > 0 \quad \text{and}$$

$$\partial \eta^{s*} / \partial \gamma = -(q^b + B + \bar{B} - S) / (q^b + B - \bar{B} + S) < 0.$$

- (ii) *With larger quoted depths, informed traders are more likely to submit market orders. That is,*

$$\partial \eta^{b*} / \partial q^a = -2\gamma(\bar{S} - B) / (\bar{S} - B - q^a - S)^2 < 0 \quad \text{and}$$

$$\partial \eta^{s*} / \partial q^b = 2\gamma(\bar{B} - S) / (q^b + B - \bar{B} + S)^2 > 0.$$

- (iii) *With more unexecuted limit-sells, informed traders are more likely to submit market orders. That is,*

$$\partial \eta^{b*} / \partial S = -2\gamma(\bar{S} - B) / (\bar{S} - B - q^a - S)^2 < 0 \quad \text{and}$$

$$\partial \eta^{s*} / \partial S = 2\gamma(q^b + B) / (q^b + B - \bar{B} + S)^2 > 0.$$

- (iv) *With more unexecuted limit-buys, informed traders are more likely to submit market orders. That is,*

$$\partial \eta^{b*} / \partial B = -2\gamma(q^a + S) / (\bar{S} - B - q^a - S)^2 < 0 \quad \text{and}$$

$$\partial \eta^{s*} / \partial B = 2\gamma(\bar{B} - S) / (q^b + B - \bar{B} + S)^2 > 0.$$

- (v) *With larger expected future market order flows, informed traders are more likely to submit limit orders. That is,*

$$\partial \eta^{b*} / \partial \bar{S} = 2\gamma(q^a + S) / (\bar{S} - B - q^a - S)^2 > 0 \quad \text{and}$$

$$\partial \eta^{s*} / \partial \bar{B} = -2\gamma(q^b + B) / (q^b + B - \bar{B} + S)^2 < 0.$$

Informed traders would avoid submitting market orders under greater quoted spreads as shown in Corollary 1(i) due to higher transaction costs. This result is supported by the empirical findings of Chung et al. (1999) and Bae et al. (2003). Corollary 1(ii) demonstrates that informed traders are more likely to place market orders with larger quoted depths because of rising execution probability. Corollary

1(iii)-(iv) imply that thicker (thinner) limit order books would lower (raise) submission rates of same-side limit orders and raise (lower) submission rates of opposite-side market orders. The reasons are obvious. If one-side limit order books are thicker, it is easier for opposite-side market orders to get executed. Thus, it is more likely for informed traders with high $|\eta|$ to submit market orders. On the other hand, since thicker limit order books imply lower execution probability for same-side limit orders, same-side limit order submission is less likely and same-side market order submission is more likely. Consequently, if thin (thick) limit order books are due to heavy (light) trading in previous periods, then we would expect larger (smaller) submission rates of same-side limit orders and smaller (larger) submission rates of opposite-side market orders in the current period. Similar submission behaviors can also be found for uninformed traders in the next section. These results are supported by Biais et al.'s (1995) empirical findings although they use data from the order-driven trading system of Pairs Bourse. Finally, Corollary 1(v) claims that if future market order flows are expected to increase (decrease), it is more (less) profitable for informed traders to submit limit orders now.

We like to point out that the results of Corollary 1(iii)-(iv) are distinct from Parlour's (1998) Proposition 1(ii), which indicates that thicker limit order books would cause lower submission rates of opposite-side market orders. In Parlour's dynamic setup, current traders' order submission activities will affect subsequent traders' order placement strategies, and current traders will take this effect into account as making submission decisions. Thus, when one-side limit order books are thicker, lower submission rates of same-side limit orders in subsequent periods appear. If the-other-side traders submit limit orders now, they are more likely to get executed in the future.

Accordingly, the submission of the-other-side market orders decreases. Nevertheless, this mechanism will not occur in our static model. Under our setup, market orders are submitted due to either valuable information or large endowment shocks not considered by Parlour (1998).

4. Uninformed Traders' Optimal Strategies

In this section, uninformed traders' optimal trading quantities using market and limit orders given endowment shocks are first derived. Then, by comparing profits acquired under the two order types, we obtain uninformed traders' optimal submission strategies. Finally, the influences of model parameters on these optimal order placement strategies are discussed.

Uninformed traders are assumed to have negative exponential utility functions. Denote by X_0 the endowment shock with a uniform distribution over $[-g, g]$ and $g > 0$. If an uninformed trader encounters shock $X_0 < (>) 0$ at time t , he will buy (sell) $|X_0|$ amount of the risky asset to rebalance his portfolio position at t . Under the circumstance, the terminal wealth of the uninformed trader at the end of period t by submitting a Δ -unit market order is

$$W_t = (X_0 + \Delta I)v + M_0 - \Delta p_t, \quad (11)$$

where p_t is the transaction price at t , and M_0 is the initial holding of the zero-return riskless asset. $I=1$ if a market-buy is submitted, and $I=-1$ if a market-sell is submitted. Since $p_t = \bar{V} + \gamma I$ by (3)-(4), (11) can be expressed as

$$W_t = W_0 + (X_0 + \Delta I)\eta - \Delta \gamma I, \quad (12)$$

where $W_0 = M_0 + X_0 \bar{V}$ is the uninformed trader's initial wealth level at the beginning of period t . Since η is normally distributed, the uninformed trader's expected utility becomes

$$\begin{aligned} E_\eta U(W_T) &= -\exp\left\{-\delta[E(W_T) - \frac{\delta}{2}\text{Var}(W_T)]\right\} \\ &= -\exp\left\{-\delta[W_0 - \Delta\gamma I - \frac{\delta}{2}(X_0 + \Delta I)^2 \sigma_\eta^2]\right\}. \end{aligned} \quad (13)$$

Maximizing (13) is equivalent to maximizing

$$U^m(\Delta | W_0) \equiv W_0 - \Delta\gamma I - \frac{\delta}{2}(X_0 + \Delta I)^2 \sigma_\eta^2. \quad (14)$$

$\Delta \leq (q^a + S)$ if a market-buy is submitted, and $\Delta \leq (q^b + B)$ if a market-sell is submitted. Here $\Delta\gamma$ represents the purchase cost or sale revenue using market orders, and $\delta(X_0 + \Delta I)^2 \sigma_\eta^2 / 2$ reflects the penalty faced by the uninformed trader when his endowment shock is not rebalanced. With more unbalanced positions, the uninformed trader's penalty would become more severe. Similar situation occurs when limit orders are submitted. By employing Lagrangian approach, we obtain that the uninformed trader's optimal trading quantities using a market-buy, b^{m*} , and a market-sell, s^{m*} , are

$$b^{m*} = \begin{cases} 0 & \text{if } X_0 \in [\frac{-\gamma}{\delta\sigma_\eta^2}, 0], \\ -\frac{\gamma}{\delta\sigma_\eta^2} - X_0 & \text{if } X_0 \in [\frac{-\gamma}{\delta\sigma_\eta^2} - q^a - S, \frac{-\gamma}{\delta\sigma_\eta^2}], \\ (q^a + S) & \text{if } X_0 \in [-g, \frac{-\gamma}{\delta\sigma_\eta^2} - q^a - S], \end{cases} \quad (15)$$

and

$$s^{m^*} = \begin{cases} 0 & \text{if } X_0 \in [0, \frac{\gamma}{\delta\sigma_\eta^2}], \\ -\frac{\gamma}{\delta\sigma_\eta^2} + X_0 & \text{if } X_0 \in [\frac{\gamma}{\delta\sigma_\eta^2}, \frac{\gamma}{\delta\sigma_\eta^2} + q^b + S], \\ (q^b + B) & \text{if } X_0 \in [\frac{\gamma}{\delta\sigma_\eta^2} + q^b + S, g], \end{cases} \quad (16)$$

respectively.

In contrast, if the uninformed trader submits a Δ -unit limit order, then his expected terminal wealth level is

$$EW_T = W_0 + [X_0 + \Delta E(\tilde{F}^\Delta)I]\eta + \Delta E(\tilde{F}^\Delta)\gamma I, \quad (17)$$

and his expected utility equals

$$\begin{aligned} E_\eta[U(EW_T)] &= -\exp\{-\delta[E_\eta[EW_T] - \frac{\delta}{2}\text{Var}(EW_T)]\} \\ &= -\exp\{-\delta[W_0 + \Delta E(\tilde{F}^\Delta)\gamma I - \frac{\delta}{2}(X_0 + \Delta E(\tilde{F}^\Delta)I)^2\sigma_\eta^2]\}, \end{aligned} \quad (18)$$

where $\Delta E(\tilde{F}^\Delta)$ is the expected trading quantity given a Δ -unit limit order is submitted by (1)-(2). Again, maximizing (18) is equivalent to maximizing:

$$U^l(\Delta E(\tilde{F}^\Delta)|W_0) \equiv W_0 + \Delta E(\tilde{F}^\Delta)\gamma I - \frac{\delta}{2}(X_0 + \Delta E(\tilde{F}^\Delta)I)^2\sigma_\eta^2. \quad (19)$$

$\Delta E(\tilde{F}^\Delta) \leq (\bar{S} - B)$ if a limit-buy is submitted and $\Delta E(\tilde{F}^\Delta) \leq (\bar{B} - S)$ if a limit-sell is submitted. $I=1$ if a limit-buy is placed, and $I=-1$ if a limit-sell is placed. According to relative magnitudes among

$\gamma/(\delta\sigma_\eta^2)$, $(\bar{S}-B)$, and $(\bar{B}-S)$, we can derive the uninformed trader's optimal limit-order submission quantities under different endowment shocks as follows. Due to the similarity, outcomes on the buy side of the market are reported here, and results on the sell side are displayed in the Appendix.

Lemma 2. (i) *When expected maximum trading volumes achievable by submitting limit-buys are large enough, uninformed traders' optimal trading quantities using limit-buys would increase with the sizes of endowment shocks for $X_0 \in [\gamma/(\delta\sigma_\eta^2) - (\bar{S}-B), 0]$. For other values of X_0 , uninformed traders will buy as many as possible. That is, if $(\bar{S}-B) > \gamma/(\delta\sigma_\eta^2)$, then b^{ls} satisfies*

$$b^{ls} E(\tilde{F}^{b^{ls}}) = \begin{cases} \frac{\gamma}{\delta\sigma_\eta^2} - X_0 & \text{if } X_0 \in [\frac{\gamma}{\delta\sigma_\eta^2} - (\bar{S}-B), 0], \\ (\bar{S}-B) & \text{if } X_0 \in [-g, \frac{\gamma}{\delta\sigma_\eta^2} - (\bar{S}-B)]. \end{cases}$$

(ii) *When expected maximum trading volumes achievable by submitting limit-buy are small, uninformed traders will buy as many as they can. That is, if $(\bar{S}-B) < \gamma/(\delta\sigma_\eta^2)$, then b^{ls} satisfies*

$$b^{ls} E(\tilde{F}^{b^{ls}}) = (\bar{S}-B) \text{ for all } X_0 \in [-g, 0].$$

Proof. See the Appendix.

Based on (15), (16), and Lemma 2, we can derive uninformed traders' optimal submission strategies on the buy side of the market, which are presented in Proposition 1 below.

Proposition 1

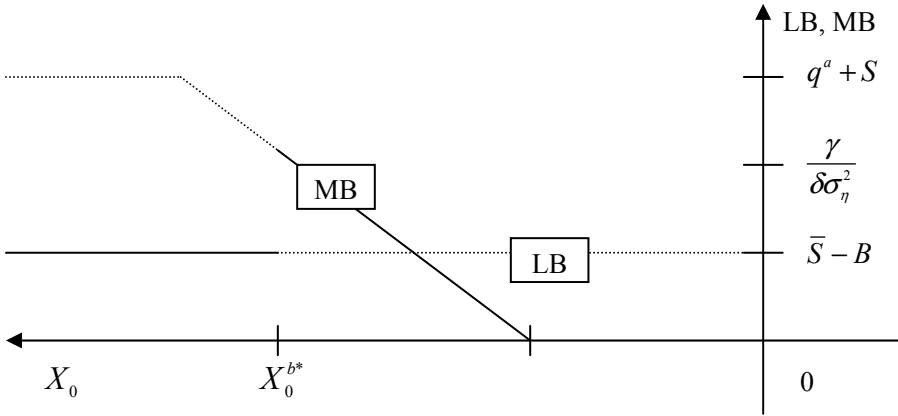
- (i) Suppose $(q^a + S) > \gamma/(\delta\sigma_\eta^2) > (\bar{S} - B) > 0$, $\gamma/(\delta\sigma_\eta^2) > (q^a + S) > (q^a + S) > (\bar{S} - B) > 0$, or $(q^a + S) > (\bar{S} - B) > \gamma/(\delta\sigma_\eta^2)$. For $X_0 \in [-g, 0]$, we have the followings.
- (ia) If $q^a + S \geq \bar{S} - B + 2\sqrt{\gamma(\bar{S} - B)/(\delta\sigma_\eta^2)} > (\bar{S} - B) > 0$, then submitting market-buys is optimal for $X_0 \leq X_0^{b*}$, and submitting limit-buys is optimal for $X_0 \geq X_0^{b*}$, where $X_0^{b*} \equiv \{-[\gamma + (\delta\sigma_\eta^2)(\bar{S} - B)] - 2\sqrt{\gamma(\delta\sigma_\eta^2)(\bar{S} - B)}\}/(\delta\sigma_\eta^2)$.
- (ib) If $0 < \bar{S} - B < q^a + S \leq \bar{S} - B + 2\sqrt{\gamma(\bar{S} - B)/(\delta\sigma_\eta^2)}$, then submitting market-buys is optimal for $X_0 \leq X_0^{b**}$, and submitting limit-buys is optimal for $X_0 \geq X_0^{b**}$, where $X_0^{b**} \equiv \gamma(q^a + S + \bar{S} - B)/[\delta\sigma_\eta^2(\bar{S} - B - q^a - S)] - (q^a + S + \bar{S} - B)/2$.
- (ii) Suppose $\gamma/(\delta\sigma_\eta^2) > (\bar{S} - B) > (q^a + S)$, $(\bar{S} - B) > (q^a + S) > \gamma/(\delta\sigma_\eta^2)$, or $(\bar{S} - B) > \gamma/(\delta\sigma_\eta^2) > (q^a + S)$. Then uninformed traders prefer limit-buys to market-buys for $X_0 \in [-g, 0]$.

Proof. See the Appendix.

The outcomes of Proposition 1(ia), 1(ib), and 1(ii) are drawn in Graphs 5-7, respectively. Proposition 1(i) shows that the optimal submission strategies of uninformed traders depend on relative magnitudes of expected maximum trading volumes achievable using limit and market orders, and sizes of endowment shocks. The intuition on the bid-side of the market is provided as follows. Given $q^a + S$ and $\bar{S} - B$, an uninformed trader with $X_0 < 0$ would obtain

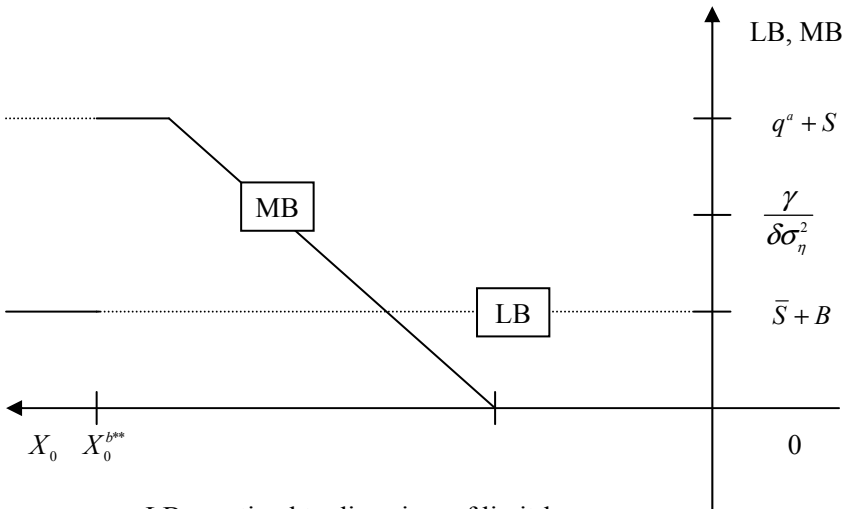
$$W_0 + \gamma(\bar{S} - B) - \frac{\delta\sigma_\eta^2}{2}(X_0 + \bar{S} - B)^2, \quad (20)$$

by (19) if limit-buys are placed, and



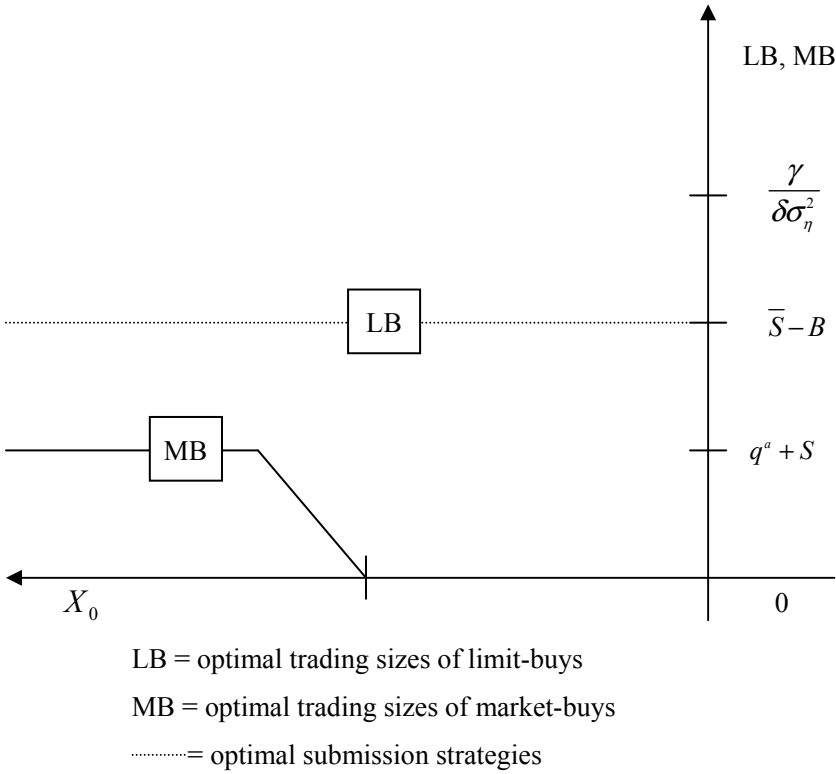
LB = optimal trading sizes of limit-buys
 MB = optimal trading sizes of market-buys
 = optimal submission strategies

Graph 5 Proposition 1(ia)



LB = optimal trading sizes of limit-buys
 MB = optimal trading sizes of market-buys
 = optimal submission strategies

Graph 6 Proposition 1(ib)



Graph 7 Proposition 1(ii)

$$W_0 - \gamma(q^a + S) - \frac{\delta\sigma_\eta^2}{2}(X_0 + q^a + S)^2, \tag{21}$$

by (14) if market-buys are placed, respectively. Again, limit-buys have the unit-price advantage. If $q^a + S < \bar{S} - B$, the unbalanced position penalty using market orders is larger than that using limit orders because $(\delta\sigma_\eta^2/2)(X_0 + q^a + S)^2 > (\delta\sigma_\eta^2/2)(X_0 + \bar{S} - B)^2$. Thus, limit-buys are the better choice for the uninformed trader whatever the value of

X_0 is. This is claimed by Proposition 1(ii). However, the story may change when $q^a + S > \bar{S} - B$. Under the latter case, the uninformed trader would face the less unbalanced position penalty by submitting market orders. Thus, when endowment shocks are large enough (i.e., $X_0 \leq X_0^{b*}$ or $X_0 \leq X_0^{b**}$), the effect of small unbalanced position penalty would outweigh the price disadvantage, hence market-buys will be submitted. Otherwise, limit-buys will still be chosen. These are the contents of Proposition 1(i).

Proposition 1 implies that market maker's quotations have significant impact on uninformed traders' optimal submission strategies. For instance, if the market maker can provide sufficient depths, uninformed traders would use market orders to satisfy their imminent financial needs. Finally, the critical points, X_0^{b*} and X_0^{b**} , are affected by quoted spreads, limit order books, and expected future market order flows. Based on Proposition 1, the smaller are $|X_0^{b*}|$ and $|X_0^{b**}|$, the more likely will market orders be submitted. Relevant results are summarized in the ensuing corollary.

Corollary 2.

- (i) *With rising quoted spreads, uninformed traders are more likely to submit limit-buys. That is,*

$$\begin{aligned} \partial X_0^{b*} / \partial \gamma &= -1 / (\delta \sigma_\eta^2) - [(\bar{S} - B) / (\delta \sigma_\eta^2)]^{1/2} < 0 \quad \text{and} \\ \partial X_0^{b**} / \partial \gamma &= (q^a + S + \bar{S} - B) / [\delta \sigma_\eta^2 (\bar{S} - B - q^a - S)] < 0. \end{aligned}$$

- (ii) *With rising quoted depths, uninformed traders are more likely to submit market-buys. That is,*

$$\partial X_0^{b**} / \partial q^a = 2\gamma(\bar{S} - B) / [\delta \sigma_\eta^2 (\bar{S} - B - q^a - S)^2] - 1/2 > 0$$

- (iii) *With increasing unexecuted limit-sells, uninformed traders are more likely to place market-buys. That is,*

$$\partial X_0^{b**} / \partial S = 2\gamma(\bar{S} - B) / [\delta \sigma_\eta^2 (\bar{S} - B - q^a - S)^2] - 1/2 > 0.$$

- (iv) *With increasing unexecuted limit-buys, uninformed traders are*

more likely to place market-buys. That is,

$$\partial X_0^{b*} / \partial B = 1 + \{\gamma / [(\bar{S} - B)(\delta\sigma_\eta^2)]\}^{1/2} > 0 \quad \text{and}$$

$$\partial X_0^{b**} / \partial B = 2\gamma(q^a + S) / [\delta\sigma_\eta^2(\bar{S} - B - q^a - S)^2] + 1/2 > 0.$$

- (v) *With rising expected future market order flows, uninformed traders are more likely to submit limit-buys. That is,*

$$\partial X_0^{b*} / \partial \bar{S} = -1 - \{\gamma / [(\bar{S} - B)(\delta\sigma_\eta^2)]\}^{1/2} < 0 \quad \text{and}$$

$$\partial X_0^{b**} / \partial \bar{S} = -2\gamma(q^a + S) / [\delta\sigma_\eta^2(\bar{S} - B - q^a - S)^2] - 1/2 < 0.$$

The results and intuitions of Corollary 2 are pretty similar to those of Corollary 1.

5. Market Maker's Optimal Quotations

In this section, optimal quoted spreads and depths of a risk-neutral monopolistic market maker are analyzed. The optimal quotations maximize the market maker's expected one-period profit given optimal order submission strategies of both informed and uninformed traders.

The market maker's expected profit function is

$$E(\pi^M) = \begin{cases} E(\pi_1^M) & \text{if } q^a + S < \bar{S} - B \text{ and } q^b + B < \bar{B} - S, \\ E(\pi_2^M) & \text{if } q^a + S < \bar{S} - B \text{ and } q^b + B > \bar{B} - S, \\ E(\pi_3^M) & \text{if } q^a + S > \bar{S} - B \text{ and } q^b + B < \bar{B} - S, \\ E(\pi_4^M) & \text{if } q^a + S > \bar{S} - B \text{ and } q^b + B > \bar{B} - S, \end{cases} \quad (22)$$

where $E(\pi_i^M), i=1, \dots, 4$, are defined below.

First, when quoted depths are small with $(q^a + S) < (\bar{S} - B)$ and $(q^b + B) < (\bar{B} - S)$, Lemma 1(i), Proposition 1(ii) and Proposition 2(ii) show that both informed and uninformed traders would submit limit orders. Then no one would trade with the market maker. Thus, the market maker's expected profit $E(\pi_1^M) = 0$.

Second, if quoted bid-depths are large enough but quoted

ask-depths are small with $(q^a + S) < (\bar{S} - B)$ and $(q^b + B) > (\bar{B} - S)$, traders may submit market-sells based on Lemma 1(iia) and Proposition 2(i). Thus, the market maker's expected profit is

$$E(\pi_2^M) = -\alpha\Psi(\eta^{s*})q^b + (1-\alpha)\gamma Eq_s^l, \quad (23)$$

where $\Psi(\eta^{s*}) = [1/(\sqrt{2\pi}\sigma_\eta)] \int_{-\infty}^{\eta^{s*}} (\eta + \gamma) \exp[-\eta^2/(2\sigma_\eta^2)] d\eta$ is the market maker's expected unit-loss when trading with an informed trader, and Eq_s^l is the market maker's expected trading quantity as transacting with an uninformed trader. According to Proposition 2(i), we have

$$Eq_s^l = \frac{1}{2g} \int_{\frac{\gamma}{\delta\sigma_\eta^2} + q^b + B}^g q^b dX_0 + \frac{1}{2g} \int_{\max\{X_0^{s*}, \frac{\gamma}{\delta\sigma_\eta^2} + B\}}^{\frac{\gamma}{\delta\sigma_\eta^2} + q^b + B} (X_0 - \frac{\gamma}{\delta\sigma_\eta^2} - B) dX_0 \quad (24)$$

if $(q^b + B) \geq \bar{B} - S + 2\sqrt{\frac{\gamma(\bar{B}-S)}{\delta\sigma_\eta^2}}$, and

$$Eq_s^l = \frac{1}{2g} \int_{X_0^{s**}}^g q^b dX_0 \quad (25)$$

if $(\bar{B} - S) < q^b + B \leq \bar{B} - S + 2\sqrt{\gamma(\bar{B} - S)/(\delta\sigma_\eta^2)}$. Equation (25) is obtained because uninformed traders would transact $(q^b + B)$ -unit market-sells for $X_0 \geq X_0^{s**}$ by Proposition 2(ib). Accordingly, the market maker would buy q^b at price p^b with probability $(g - X_0^{s**})/(2g)$. Conversely, when $q^b + B \geq \bar{B} - S + 2\sqrt{\gamma(\bar{B} - S)/(\delta\sigma_\eta^2)}$, the market maker may transact with an uninformed trader having $X_0 \geq X_0^{s*}$ by Proposition 2(ia). And the uninformed with $X_0 \in [\gamma/(\delta\sigma_\eta^2), \gamma/(\delta\sigma_\eta^2) + q^b + B]$ would submit an $[X_0 - \gamma/(\delta\sigma_\eta^2)]$ -unit market-sell by (16). Then, the realized trading quantity depends on

relative magnitudes of X_0^{s*} and $[\gamma/(\delta\sigma_\eta^2)+B]$.

If $X_0^{s*} \geq [\gamma/(\delta\sigma_\eta^2)+B]$, the market-sell quantity demanded by uninformed traders with $X_0 \in [X_0^{s*}, \gamma/(\delta\sigma_\eta^2)+q^b+B]$ cannot be satisfied by the sum of limit-buy-book amounts and quoted bid-depths. Since limit order books have execution priority over market maker's quotations, the realized trading quantity between the market maker and uninformed traders is $[X_0 - \gamma/(\delta\sigma_\eta^2) - B]$. On the other hand, uninformed traders having $X_0 \in [\gamma/(\delta\sigma_\eta^2)+q^b+B, g]$ would trade q^b units with the market maker due to the quantity constraint. If $X_0^{s*} \leq [\gamma/(\delta\sigma_\eta^2)+B]$, then the market maker would never trade with uninformed traders having $X_0 \in [X_0^{s*}, \gamma/(\delta\sigma_\eta^2)+B]$ even market-sells are submitted. That is because the market-sell quantity is relatively small compared to the amount contained in all unexecuted limit-buys, i.e., $X_0 - \gamma/(\delta\sigma_\eta^2) \leq B$. However, uninformed traders having $X_0 \in [\gamma/(\delta\sigma_\eta^2)+B, \gamma/(\delta\sigma_\eta^2)+q^b+B]$ would trade $[X_0 - \gamma/(\delta\sigma_\eta^2) - B]$ units with the market maker. As for uninformed traders having $X_0 \geq \gamma/(\delta\sigma_\eta^2)+q^b+B$, they can only buy q^b units from the market maker. Thus, we obtain (24).

Similarly, when $(q^a+S) > (\bar{S}-B)$ and $(q^b+B) < (\bar{B}-S)$, the market maker's expected profit is:

$$E(\pi_3^M) = -\alpha\Psi(\eta^{b*})q^a + (1-\alpha)\gamma Eq_b^l, \quad (26)$$

where $\Psi(\eta^{b*}) = \frac{1}{\sqrt{2\pi}\sigma_\eta} \int_{\eta^{b*}}^{\infty} (\eta - \gamma) \exp(-\frac{\eta^2}{2\sigma_\eta^2}) d\eta$,

$$Eq_b^l = \frac{1}{2g} \int_{-g}^{-\frac{r}{\delta\sigma_\eta^2} - (q^a+S)} q^a dX_0 + \frac{1}{2g} \int_{-\frac{r}{\delta\sigma_\eta^2} - (q^a+S)}^{\min\{X_0^{b*}, -\frac{\gamma}{\delta\sigma_\eta^2} - S\}} (-X_0 - \frac{\gamma}{\delta\sigma_\eta^2} - S) dX_0 \quad (27)$$

if $q^a + S \geq \bar{S} - B + 2\sqrt{\gamma(\bar{S} - B)/(\delta\sigma_\eta^2)}$, and

$$Eq_b^l = \frac{1}{2g} \int_{-g}^{X_0^{j^{**}}} (q^a) dX_0 \quad (28)$$

if $(\bar{S} - B) < (q^a + S) \leq \bar{S} - B + 2\sqrt{\gamma(\bar{S} - B)/(\delta\sigma_\eta^2)}$.

Finally, when quoted depths are large enough with $(q^a + S) > (\bar{S} - B)$ and $(q^b + B) > (\bar{B} - S)$, the market maker may trade on both sides of the market with informed and uninformed traders. Thus, his expected profit is

$$E(\pi_4^M) = E(\pi_2^M) + E(\pi_3^M). \quad (29)$$

Equilibrium spreads and depths are pairs of (γ, q^a, q^b) , which maximize the market maker's expected profit function $E(\pi^M)$. Since $E(\pi^M)$ is very complicated, no close-form solutions can be derived. Numerical solutions hence are presented and analyzed in the next section.

6. Simulation Results

In this section, numerical derivations of equilibrium quotations are first described. Then, equilibrium solutions and the corresponding comparative statics results are inspected.

Based on relative magnitudes of model's trading-environment parameters and market maker's quotations, fifteen disjoint areas with

equilibrium existence potential are divided.⁹ *Mathematica* software is employed to acquire equilibrium spreads and depths. The procedures of finding equilibrium spreads and depths are as follows. First, pair points of spreads and depths satisfying the corresponding first-order conditions and required assumptions in each area are uncovered. Second, to make sure that these spreads and depths are potential equilibria, market maker's expected profits at these points and their nearby points are compared. Saddle points will be deleted. Third, the left potential equilibrium points are then plugged into the associated expected profit functions to discover those maximizing expected profits. After numerous trials by setting different parameter values, it is found that the equilibrium quotations occur only in the following three areas. And given specific values of all parameters, our solutions, if they exist, are unique.

$$A = \{(\gamma, q^a, q^b) \mid q^a + S \geq \bar{S} - B + 2\sqrt{\frac{\gamma(\bar{S} - B)}{\delta\sigma_\eta^2}}, (q^b + B) \geq \bar{B} - S + 2\sqrt{\frac{\gamma(\bar{B} - S)}{\delta\sigma_\eta^2}}; X_0^{b*} \leq \frac{-\gamma}{\delta\sigma_\eta^2} - S, \text{ and } X_0^{s*} \geq \frac{\gamma}{\delta\sigma_\eta^2} + B\},$$

$$E = \{(\gamma, q^a, q^b) \mid (q^a + S) < (\bar{S} - B), q^b + B \geq \bar{B} - S + 2\sqrt{\frac{\gamma(\bar{B} - S)}{\delta\sigma_\eta^2}}, \text{ and } X_0^{s*} \geq \frac{\gamma}{\delta\sigma_\eta^2} + B\},$$

⁹ These disjoint areas are obtained by combining the following comparison results. And the comparison is conducted by checking relative variables magnitudes in four sets: $q^a + S$ and $\bar{S} - B + 2\sqrt{\gamma(\bar{S} - B)/(\delta\sigma_\eta^2)}$, $q^b + B$ and $\bar{B} - S + 2\sqrt{\gamma(\bar{B} - S)/(\delta\sigma_\eta^2)}$, X_0^{b*} and $-\gamma/(\delta\sigma_\eta^2) - S$ and X_0^{s*} and $\gamma/(\delta\sigma_\eta^2) + B$.

$$F = \{(\gamma, q^a, q^b) \mid (q^a + S \geq \bar{S} - B + 2\sqrt{\frac{\gamma(\bar{S} - B)}{\delta\sigma_\eta^2}}, (q^b + B) < (\bar{B} - S), \text{ and}$$

$$X_0^{b*} \leq -\frac{\gamma}{\delta\sigma_\eta^2} - S\}.$$

Under the quotations in area A , all kinds of orders can be submitted by investors. Under the quotations in area E , all kinds of orders except for market-buys can be submitted by investors based on Lemma 1(iia), Proposition 1(ii), and Proposition 2(i). And under the quotations in area F , all kinds of orders except for market-sells can be submitted by investors based on Lemma 1(iib), Proposition 1(i) and Proposition 2(ii).

Table 1 displays areas and parameter conditions of equilibrium quotations. It is observed that equilibrium spreads and depths would occur in area E when expected future market-sell flows, \bar{S} , are greater than expected future market-buy flows, \bar{B} , to a certain degree; and unexecuted limit-sells, S , are larger than unexecuted limit-buys, B . Conversely, equilibrium spreads and depths would occur in area F when $\bar{B} > \bar{S}$, $B > S$, and $\bar{B} - \bar{S}$ is large enough. Otherwise, market maker's optimal quotations would occur in the A area. This means that in certain circumstances the market maker will trade merely on one side of the market by quoting small depths. Kavajecz (1999) discovers these phenomena empirically without describing the pre-conditions for them. The pre-conditions for equilibria occurring in area E include thinner limit-buy books, and large enough expected future market-sell flows. Under the conditions, the market maker is more likely to trade with informed traders on the ask side, which is shown later. Thus, the market maker will pass unwanted trades to limit order books, and provide liquidity on the more illiquid bid side by quoting large enough depths. Opposite situations occur in the F area. All explanations for equilibrium

locations are given below.

Suppose $\bar{S} > \bar{B}$, $S > B$, and $\bar{S} - \bar{B}$ is large enough. Given large future market-sell flows and thin limit-buy books, buy-side traders are likely to submit limit instead of market orders due to their higher expected execution rates. However, whether submission of limit-buys would result in equilibrium depends on market maker's quotations. If the market maker quotes a relatively small spread, γ , and a sufficiently large ask-depth, q^a , then he has chances to trade with high-information-return (i.e., high η) or big-liquidity-shock investors, who would submit market-buys by Lemma 1(iib) and Proposition 1. Facing smaller γ and larger q^a , traders are likely to submit more market-buys by Corollaries 1(ii) and 2(i). On the other hand, with greater expected future market-sell flows, traders are likely to submit fewer market-buys by Corollaries 1(v) and 2(ii). Relative magnitudes of the two effects would determine the type of traders that are more likely to submit market orders. The probability of uninformed traders' submitting market-buys decreases faster than that of informed traders' market-buys as \bar{S} increases. And the probability of informed traders' market-buys increase faster than that of uninformed traders' submitting market-buys as q^a increases.¹⁰ Thus for a sufficiently large \bar{S} , the market maker may earn negative expected payoff if he transacts merely on the ask side due to the high probability of meeting an informed trader. Consequently, the market maker can avoid unwanted trades by

¹⁰ From Corollaries 1 and 2, we have $\partial\eta^{b^*}/\partial\bar{S} > 0$, $\partial\eta^{b^*}/\partial q^a < 0$, $\partial X_0^{b^*}/\partial\bar{S} < 0$, $\partial X_0^{b^*}/\partial q^a = 0$. By simple calculation, we obtain $\partial^2\eta^{b^*}/\partial\bar{S}^2 = -4\gamma(q^a + S)/(\bar{S} - B - q^a - S)^3 > 0$, $\partial^2\eta^{b^*}/\partial(q^a)^2 = -4\gamma(\bar{S} - B)/(\bar{S} - B - q^a - S)^3 > 0$, $\partial^2 X_0^{b^*}/\partial\bar{S}^2 = (1/2)[\gamma/(\delta\sigma_\eta^2)]^{1/2}[1/(\bar{S} - B)]^{3/2} > 0$, and $\partial^2 X_0^{b^*}/\partial(q^a)^2 = 0$. It is easy to see that $\partial^2\eta^{b^*}/\partial\bar{S}^2 < \partial^2 X_0^{b^*}/\partial\bar{S}^2$ for large enough \bar{S} , and $\partial^2\eta^{b^*}/\partial(q^a)^2 > \partial^2 X_0^{b^*}/\partial(q^a)^2$.

Table 1 Areas and Parameter Conditions of Equilibrium Quotations*

Area	γ	q^a	q^b	Profit
$\bar{S}=14 \quad \bar{B}=10 \quad S=2 \quad B=5$				
<i>A</i>	0.93395	20.2687	17.0038	1.77664**
<i>E</i>	1.02803		17.2689	0.91454
<i>F</i>	0.86357	19.9212		0.87642
$\bar{S}=14 \quad \bar{B}=10 \quad S=8 \quad B=5$				
<i>A</i>	1.43160	13.8180	16.6104	2.34903
<i>E</i>	2.58131		14.2636	2.44203**
<i>F</i>	0.87412	14.2235		0.58739
$\bar{S}=14 \quad \bar{B}=10 \quad S=5 \quad B=2$				
<i>A</i>	0.90633	17.9372	18.7903	1.61799
<i>E</i>	1.64716		19.3188	1.97955**
<i>F</i>	0.45914	13.9293		0.29512
$\bar{S}=14 \quad \bar{B}=10 \quad S=5 \quad B=8$				
<i>A</i>	1.51288	16.6138	13.5916	2.53949**
<i>E</i>	1.65804		13.3015	1.21073
<i>F</i>	1.41502	16.7963		1.34387
$\bar{S}=14 \quad \bar{B}=10 \quad S=5 \quad B=5$				
<i>A</i>	1.16825	17.3579	16.8746	2.10895**
<i>E</i>	1.64134		16.3129	1.59512
<i>F</i>	0.86783	17.0695		0.73173
$\bar{S}=12 \quad \bar{B}=10 \quad S=2 \quad B=5$				
<i>A</i>	1.11871	20.1043	17.3140	2.25867**
<i>E</i>	1.02803		17.2689	0.91454
<i>F</i>	1.20256	20.1180		1.35700
$\bar{S}=12 \quad \bar{B}=10 \quad S=8 \quad B=5$				
<i>A</i>	1.73660	13.1406	16.0947	2.93163**
<i>E</i>	2.58131		14.2636	2.44203
<i>F</i>	1.21842	14.1551		0.87822

Table 1 Areas and Parameter Conditions of Equilibrium Quotations*
(Continued.)

Area	γ	q^a	q^b	Profit
$\bar{S}=12$ $\bar{B}=10$ $S=5$ $B=2$				
<i>A</i>	1.10396	17.5148	19.7091	2.19144**
<i>E</i>	1.64716		19.3188	1.97955
<i>F</i>	0.72261	16.4175		0.56656
$\bar{S}=12$ $\bar{B}=10$ $S=5$ $B=8$				
<i>A</i>	1.79840	15.9913	12.9998	3.06510**
<i>E</i>	1.65804		13.3015	1.21073
<i>F</i>	1.92279	15.7227		1.86980
$\bar{S}=12$ $\bar{B}=10$ $S=5$ $B=5$				
<i>A</i>	1.40237	16.8566	16.7617	2.65113**
<i>E</i>	1.65134		16.3129	1.59512
<i>F</i>	1.20883	17.1396		1.11758
$\bar{S}=10$ $\bar{B}=10$ $S=2$ $B=5$				
<i>A</i>	1.31366	19.8315	17.0567	2.75174**
<i>E</i>	1.02803		17.2689	0.91454
<i>F</i>	1.64716	19.3188		1.97955
$\bar{S}=10$ $\bar{B}=10$ $S=8$ $B=5$				
<i>A</i>	2.07135	12.3968	15.3903	3.49813**
<i>E</i>	2.58131		14.2636	2.44203
<i>F</i>	1.65804	13.3015		1.21073
$\bar{S}=10$ $\bar{B}=10$ $S=5$ $B=2$				
<i>A</i>	1.31366	17.0567	19.8315	2.75174**
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<i>F</i>	1.02803	17.2689		0.91454

Table 1 Areas and Parameter Conditions of Equilibrium Quotations*
 (Continued.)

Area	γ	q^a	q^b	Profit
$\bar{S}=10$ $\bar{B}=10$ $S=5$ $B=8$				
<i>A</i>	2.07135	15.3903	12.3968	3.49813**
<i>E</i>	1.65804		13.3015	1.21073
<i>F</i>	2.58131	14.2636		2.44203
$\bar{S}=10$ $\bar{B}=10$ $S=5$ $B=5$				
<i>A</i>	1.65134	16.3129	16.3129	3.19024**
<i>E</i>	1.64134		16.3129	1.59512
<i>F</i>	1.65134	16.3129		1.59512
$\bar{S}=10$ $\bar{B}=12$ $S=2$ $B=5$				
<i>A</i>	1.10396	19.7091	17.5148	2.19144**
<i>E</i>	0.72261		16.4175	0.56656
<i>F</i>	1.64716	19.3188		1.97955
$\bar{S}=10$ $\bar{B}=12$ $S=8$ $B=5$				
<i>A</i>	0.79840	12.9998	15.9913	3.06510**
<i>E</i>	1.92279		15.7227	1.86980
<i>F</i>	1.65804	13.3015		1.21073
$\bar{S}=10$ $\bar{B}=12$ $S=5$ $B=2$				
<i>A</i>	1.11871	17.3140	20.1043	2.25867**
<i>E</i>	1.20256		20.1180	1.35705
<i>F</i>	1.02803	17.2689		0.91454
$\bar{S}=10$ $\bar{B}=12$ $S=5$ $B=8$				
<i>A</i>	1.73660	16.0947	13.1406	2.93163**
<i>E</i>	1.21842		14.1551	0.87822
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Table 1 Areas and Parameter Conditions of Equilibrium Quotations*
(Continued.)

Area	γ	q^a	q^b	Profit
$\bar{S}=10$ $\bar{B}=12$ $S=5$ $B=5$				
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$\bar{S}=10$ $\bar{B}=14$ $S=2$ $B=5$				
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$\bar{S}=10$ $\bar{B}=14$ $S=8$ $B=5$				
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$\bar{S}=10$ $\bar{B}=14$ $S=5$ $B=2$				
<i>A</i>	0.93395	17.0038	20.2687	1.77667**
<i>E</i>	0.86357		19.9212	0.87642
<i>F</i>	1.02803	17.2689		0.91454
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<i>A</i>	1.43160	16.6104	13.8180	2.34903
<i>E</i>	0.87412		14.2235	0.58739
<i>F</i>	2.58131	14.2636		2.44203**
$\bar{S}=10$ $\bar{B}=14$ $S=5$ $B=5$				
<i>A</i>	1.16825	16.8746	17.3579	2.10895**
<i>E</i>	0.86783		17.0695	0.73173
<i>F</i>	1.65134	16.3129		1.59512

Notes: 1.*: $\alpha=0.5$, $\sigma_\eta^2=0.5$, $\delta=0.9$, $g=25$.

2.**: Equilibrium quotations.

quoting a small q^a with $q^a + S < \bar{S} - B$. This means that equilibrium would not occur in area A. Under the circumstances, both informed and uninformed traders would submit limit-buys based on Lemma 1(ii) and Proposition 1(ii), and the market maker would find no trading opponents on the ask side and hence lose nothing. On the other hand, given small \bar{B} and large S , sell-side investors have less incentive to submit limit orders. Then, the market maker could raise bid depths, q^b , to attract the submission of market-sells, and increase quoted spreads to reduce the loss when trading with informed traders. Therefore, equilibrium quotations may occur in the E area.

By similar arguments, equilibrium quotations may occur in area F when $\bar{B} > \bar{S}$, $B > S$, and $\bar{B} - \bar{S}$ is large enough. Under other market scenarios, investors would have no advantages by submitting limit orders. Consequently, the market maker will trade on both sides of the market by providing enough depths. Equilibrium spreads and depths thus would occur in the A area.

According to locations of the equilibrium quotations, comparative statics analyses are conducted. We investigate the comovements of equilibrium spreads and depths in the three areas corresponding to changing trading-environment parameters, such as degree of adverse selection (α), variance of private information (σ_n^2), uninformed trader's risk aversion level (δ), magnitude of endowment shocks (g), thickness of limit order books (B , or S), and expected future market order flows (\bar{B} , or \bar{S}). Figures 1a-1g and Figures 2a-2f show the moving directions of equilibrium quotations with changing trading-environment parameters in areas A and E , respectively. The numerical data behind all figures are available on request. Sizes of equilibrium spreads are enlarged 10 times in all figures to make the comovements of equilibrium quotations more easily seen. Because, by

employing symmetric parameter values, the bid-side trading results in area E and the ask-side trading results in area F are the same, the simulation outcomes for area F are omitted.

First, as the ratio of informed traders (α) increases, the market maker would slightly raise spreads and slightly lower depths to protect himself from the rising risk of adverse selection. The rising spreads would increase the transaction cost of market orders, and the falling depths would lower the trading probability of the market maker with informed traders. This outcome is the same as those of Kavajecz (1998), Chen et al. (2000), and Dupont (2000). Our numerical results show that changing percentages of depths are a bit larger than those of spreads in all areas although changing magnitudes of equilibrium quotations are quite small. It seems that the market maker prefers reducing depths to enhancing spreads in avoiding trading loss to informed traders.

Second, for relatively small σ_η^2 both equilibrium spreads and depths increase significantly as σ_η^2 increases. However, for relatively large σ_η^2 only equilibrium spreads increase significantly with rising σ_η^2 . Larger variance of private information means that market maker's loss to informed traders would increase, and the transaction cost of market orders would rise. Hence, the market maker corresponds by widening quoted spreads, and knows that limit orders are in turn preferred by his trading opponents. Motivated by higher revenue, the market maker would raise quoted depths at the same time to lure the submission of market orders. Therefore, equilibrium spreads and depths move in the same direction. This finding is consistent with the empirical result of Liu et al. (2001). Chen et al. (2000) reach a similar conclusion given market orders can be employed only. According to Black (1971) and Kyle (1985), spreads and depths are important indices to measure market liquidity. Thus, our uni-directional movements of equilibrium spreads and depths leave the impact of changing

σ_η^2 on market maker's liquidity provision inconclusive. Moreover, our outcome implies that the market maker would offset the adverse effect of varying σ_η^2 by changing both spreads and depths when σ_η^2 is relatively small, and changing spreads merely when σ_η^2 is relatively large.

Third, equilibrium spreads and depths will increase significantly and slightly, respectively, when δ rises. As uninformed traders become more risk averse, their hedging desires become stronger under given endowment shocks. The market maker could provide deeper depths to absorb the increasing liquidity-motivated transactions. However, greater depths would raise his loss to informed traders. Thus, he would widen quoted spreads simultaneously to control the damage. In other words, equilibrium spreads and depths would move in the same direction. This finding is absent in Kavajecz's (1998) and Dupont's (2000) studies since traders are all assumed risk-neutral by them.

Fourth, as magnitudes of liquidity shocks increase, the market maker has bigger chance to trade with uninformed traders at large quantities. Thus, it is optimal for him to raise quoted depths. However, to lower the probability of encountering informed traders, the market maker should strategically increase quoted spreads too. Hence, equilibrium spreads and depths vary uni-directionally. While the finding about the impact of changing endowment shocks on spreads and bid depths here is the same as Kavajecz's (1998), the result about the impact of changing endowment shocks on ask depths is different. Again, the influence of liquidity shock sizes on market maker's liquidity provision is inconclusive.

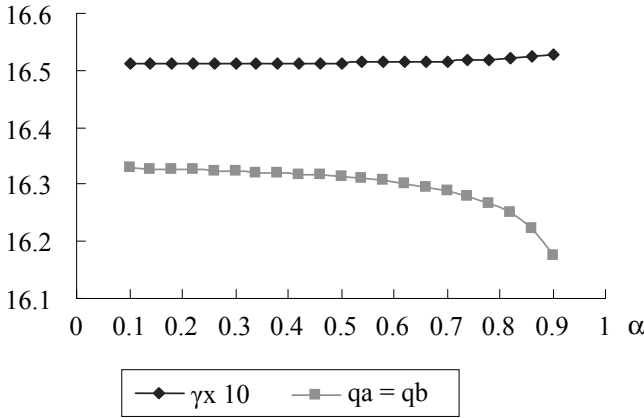
Finally, the effects of changing thickness of limit order books and varying magnitudes of expected future market order flows on equilibrium quotations are new findings of this study. As expected future market order flows increase, investors would prefer submitting

limit orders. To raise trading opportunities under the circumstance, the market maker would narrow quoted spreads to reduce the transaction cost of market orders as well as increase quoted depths to enhance the benefit of submitting market orders. Accordingly, equilibrium spreads and depths move in opposite directions.

When limit-buy or limit-sell books become thicker, the market maker would increase spreads and decrease depths if equilibria are in area A . Since large unexecuted limit-buys (limit-sells) lower these orders' future execution chances, market-buys (market-sells) would be submitted. The market maker hence decreases depths to avoid encountering informed traders, and raise spreads to compensate for loss due to low trading quantities. In area E (F), bid-depths (ask-depths) decrease significantly, but equilibrium spreads decrease slightly as limit-buy (limit-sell) books become thicker. The decreasing depths will lower market maker's loss when market-sells (market-buys) are submitted by informed traders. Again it shows that the market maker prefer reducing depths to spreads in avoiding unwanted trades.

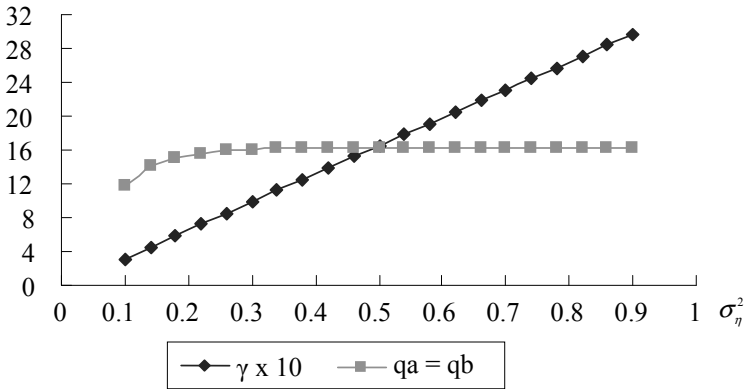
7. Extensions

In Sections 3-6, traders are not allowed to submit aggressive limit orders and the execution probability of limit orders is exogenous. These assumptions are relaxed now. It is discovered that the results in Sections 3-5 and the outcome that the market maker may stay on one side of the market in Section 6 remain true qualitatively. However, to answer whether comparative statics results in Section 6 hold, further research is needed. Moreover, we find that more aggressive the limit orders are, the more likely the market maker would conduct one-side trading. In the following, the new setup is demonstrated and the intuition for the outcomes on the buy side of the market are provided. All proofs are available on request.



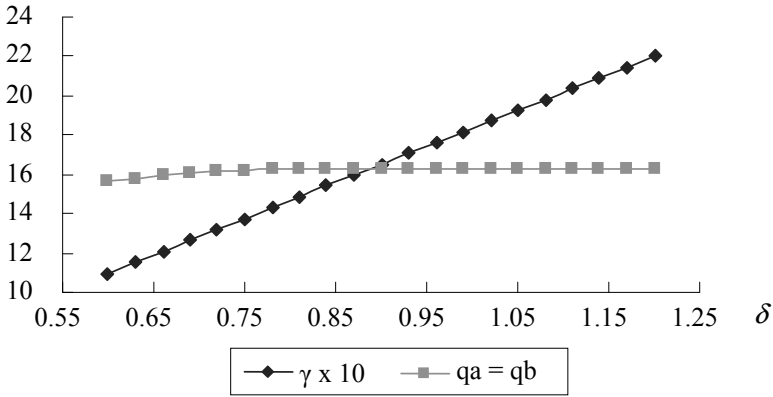
Note: $\sigma_{\eta}^2 = 0.5, \delta = 0.9, g = 25, \bar{B} = \bar{S} = 10, B = S = 5.$

Figure 1a Simulated Results of Changing Ratios of Informed Traders for *A*-area Equilibria



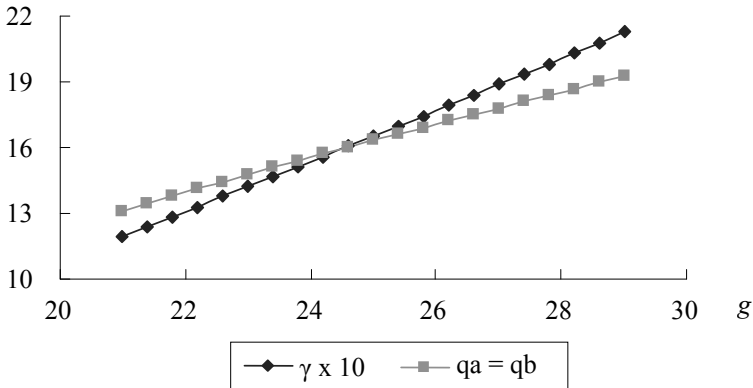
Note: $\alpha = 0.5, \delta = 0.9, g = 25, \bar{B} = \bar{S} = 10, B = S = 5.$

Figure 1b Simulated Results of Changing Variances of Private Information for *A*-area Equilibria



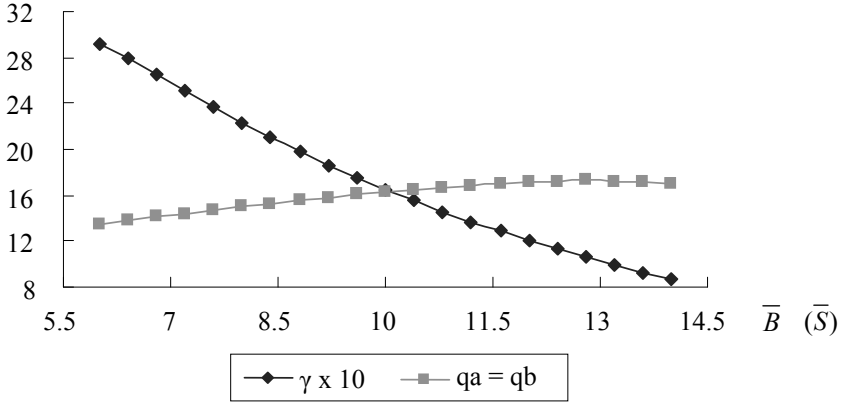
Note: $\alpha = 0.5, \sigma_\eta^2 = 0.5, g = 25, \bar{B} = \bar{S} = 10, B = S = 5.$

Figure 1c Simulated Results of Changing Risk Averse Levels of Uninformed Traders for *A*-area Equilibria



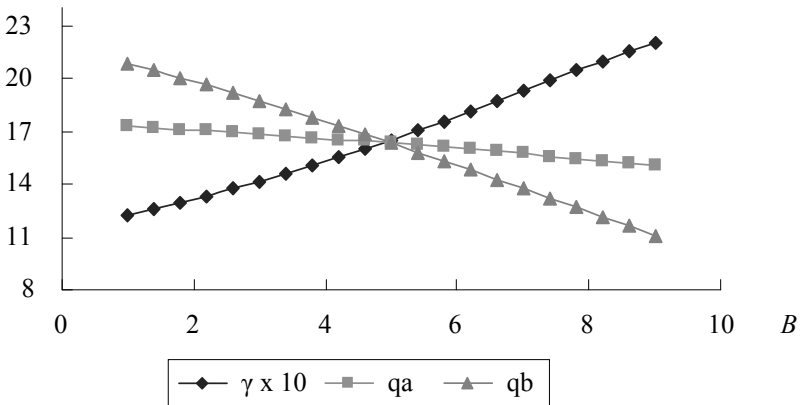
Note: $\alpha = 0.5, \sigma_\eta^2 = 0.5, \delta = 0.9, \bar{B} = \bar{S} = 10, B = S = 5.$

Figure 1d Simulated Results of Changing Magnitudes of Liquidity Shocks for *A*-area Equilibria



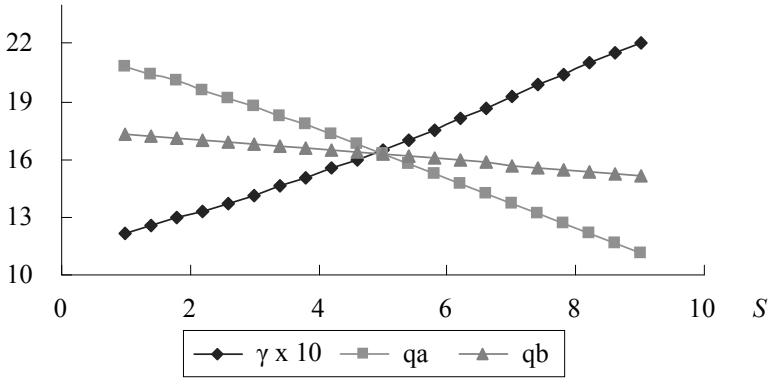
Note: $\alpha = 0.5, \sigma_\eta^2 = 0.5, \delta = 0.9, g = 25, B = S = 5$.

Figure 1e Simulated Results of Changing Sizes of Expected Future Market Order Flows for *A*-area Equilibria



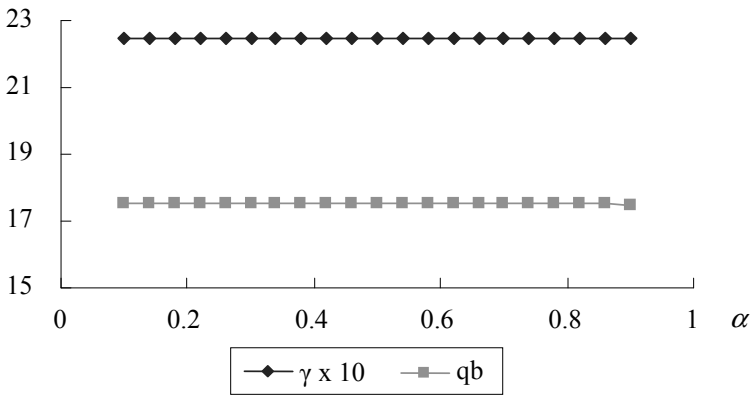
Note: $\alpha = 0.5, \sigma_\eta^2 = 0.5, \delta = 0.9, g = 25, \bar{B} = \bar{S} = 10, S = 5$.

Figure 1f Simulated Results of Changing Thickness of Limit Buy Books for *A*-area Equilibria



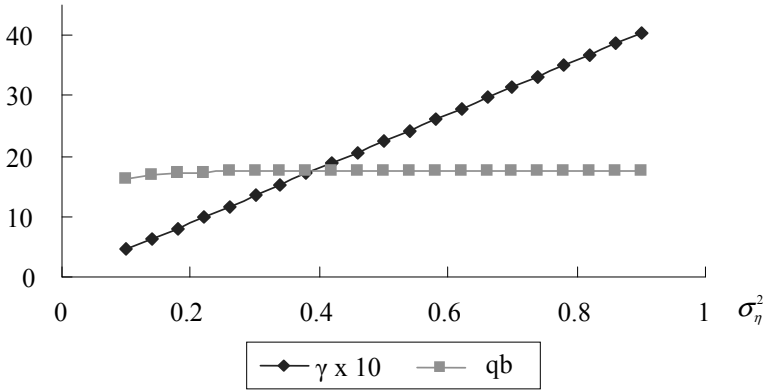
Note: $\alpha = 0.5, \sigma_{\eta}^2 = 0.5, \delta = 0.9, g = 25, \bar{B} = \bar{S} = 10, B = 5$.

Figure 1g Simulated Results of Changing Thickness of Limit Sell Books for A-area Equilibria



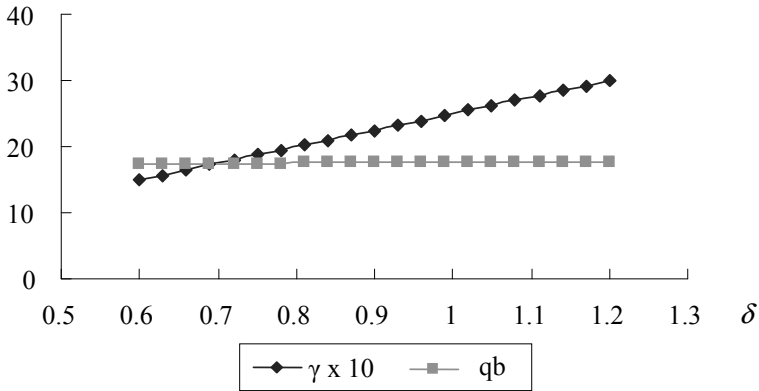
Note: $\sigma_{\eta}^2 = 0.5, \delta = 0.9, g = 25, \bar{B} = 8, \bar{S} = 14, B = 2.5, S = 5$.

Figure 2a Simulated Results of Changing Ratios of Informed Traders for E-area Equilibria



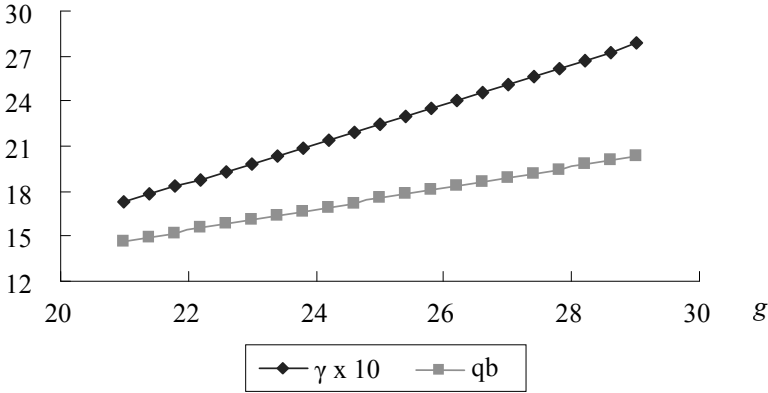
Note: $\alpha = 0.5, \delta = 0.9, g = 25, \bar{B} = 8, \bar{S} = 14, B = 2.5, S = 5.$

Figure 2b Simulated Results of Changing Variances of Private Information for *E*-area Equilibria



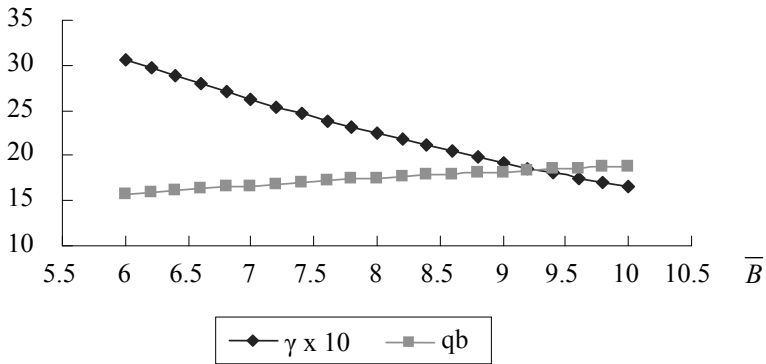
Note: $\alpha = 0.5, \sigma_\eta^2 = 0.5, g = 25, \bar{B} = 8, \bar{S} = 14, B = 2.5, S = 5.$

Figure 2c Simulated Results of Changing Risk Averse Levels of Uninformed Traders for *E*-area Equilibria



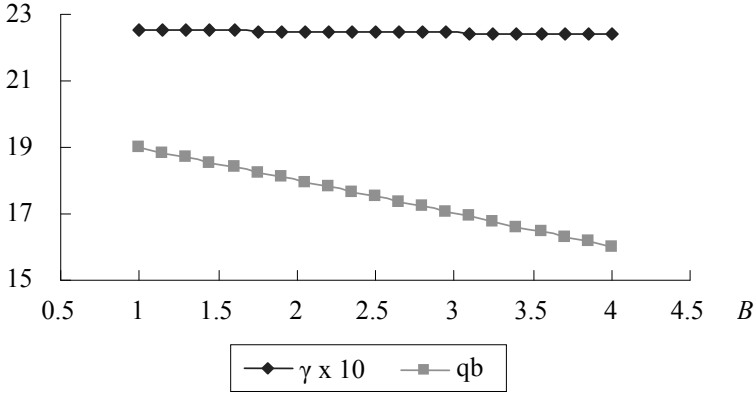
Note: $\alpha = 0.5, \sigma_{\eta}^2 = 0.5, \delta = 0.9, \bar{B} = 8, \bar{S} = 14, B = 2.5, S = 5.$

Figure 2d Simulated Results of Changing Magnitudes of Liquidity Shocks for *E*-area Equilibria



Note: $\alpha = 0.5, \sigma_{\eta}^2 = 0.5, \delta = 0.9, g = 25, \bar{S} = 14, B = 2.5, S = 5.$

Figure 2e Simulated Results of Changing Sizes of Expected Future Market Buy Flows for *E*-area Equilibria



Note: $\alpha = 0.5, \sigma_\eta^2 = 0.5, \delta = 0.9, g = 25, \bar{B} = 8, \bar{S} = 14, S = 5$.

Figure 2f Simulated Results of Changing Thickness of Limit Buy Books for *E*-area Equilibria

Let $p^b - \beta$ be the price of a limit-buy submitted by traders. The set of all possible values for β is $\{-\gamma, -(n-1)\gamma/n, \dots, 0\}$ and n is a positive integer. The larger $|\beta|$ is, the more aggressive the limit order is. Denote by $\bar{S}(\beta)$ the expected future market-sell flows given β . It is plausible to assume $\partial\bar{S}(\beta)/\partial\beta < 0$ because the larger market sell flow is expected to arrive after time t with higher aggressive degree of the limit order. Let $B(\beta)$ be the unexecuted limit-buy quantities at prices greater than or equal to $(p^b - \beta)$ up to time $t-1$. Suppose $\partial B(\beta)/\partial\beta > 0$, which means that smaller amount of unexecuted limit-buys would be left as prices of the limit-buys increase. Finally, let $E(\tilde{F}^{bt} | \beta)$ be the expected execution rates when the limit-buy is priced at $(p^b - \beta)$. It is reasonable to assume that $\partial E(\tilde{F}^{bt} | \beta)/\partial\beta < 0$ since the higher execution rate is expected with rising aggressive degree of the limit order. Accordingly, an informed trader will choose the optimal

price and quantity of the limit order, (β^*, b_l^*) , to maximize his expected profit, $b_l^* E(\tilde{F}^{b_l^*} | \beta)(\eta + \gamma + \beta)$, given the constraint of $b_l^* E(\tilde{F}^{b_l^*} | \beta) \leq \bar{S}(\beta) - B(\beta)$.¹¹

Based on the above setup, the expected profits of the informed trader with $\eta > 0$ by submitting limit-buys and market-buys are

$$(\eta + \gamma + \beta^*)[\bar{S}(\beta^*) - B(\beta^*)] \text{ and } (\eta - \gamma)(q^a + S),$$

respectively. Since $\beta^* \geq -\gamma$, we have $\eta + \gamma + \beta^* > \eta - \gamma > 0$, which means that submitting limit-buys still has the unit-price advantage. Thus, the arguments following Lemma 1 can be applied here. Hence, if $q^a + S < \bar{S}(\beta^*) - B(\beta^*)$, it is optimal for informed traders to submit limit-buys for all $\eta > 0$. Conversely, if $q^a + S > \bar{S}(\beta^*) - B(\beta^*)$, then informed traders with good enough private information will submit market orders. Otherwise, limit-buys will be the better choice. Thus, Lemma 1 remains true qualitatively, so does Corollary 1.

Similarly, by submitting market-buys and limit-buys at price $p^b - \beta^*$, uninformed traders with $X_0 < 0$ would obtain

$$W_0 - \gamma(q^a + S) - \frac{\gamma\sigma_\eta^2}{2}(X_0 + q^a + S)^2 \text{ and}$$

$$W_0 + (\gamma + \beta^*)[\bar{S}(\beta^*) - B(\beta^*)] - \frac{\gamma\sigma_\eta^2}{2}[X_0 + \bar{S}(\beta^*) - B(\beta^*)]^2,$$

respectively.¹² Same arguments following Proposition 1 can be applied here. Accordingly, Propositions 1-2 would remain true qualitatively, so

¹¹ By the intermediate value theorem, β^* exists if $\partial^2 \bar{S}(\beta) / \partial \beta^2 - \partial^2 B(\beta) / \partial \beta^2 < 0$, $\bar{S}(0) - B(0) > 0$, and $\partial \bar{S}(0) / \partial \beta - \partial B(0) / \partial \beta < \bar{S}(0) - B(0)$. Moreover, (β^*, b_l^*) satisfies the condition of $b_l^* E(\tilde{F}^{b_l^*} | \beta^*) = \bar{S}(\beta^*) - B(\beta^*)$.

¹² Here, the existence of β^* is implied by $\partial \bar{S}(\beta) / \partial \beta - \partial B(\beta) / \partial \beta < 0$.

does Corollary 2.

Finally, as in Section 6, the market maker will stick on the bid side of the market when $\bar{S}(\beta^*) > \bar{B}$, $S > B(\beta^*)$, and $\bar{S}(\beta^*) - \bar{B}$ is large enough, where \bar{B} is the expected future market-buy flows.¹³ Moreover, since $\partial \bar{S}(\beta) / \partial \beta < 0$ and $\partial B(\beta) / \partial \beta > 0$, the three conditions are more likely to be met when more aggressive limit orders are submitted. Hence, the market maker is more likely to do one-side trading as facing more aggressive limit orders.

8. Comparing Our Major Findings with Those of Previous Studies

To emphasize the contributions of this paper, this section first summarize the empirically testable findings from our numerical outcomes. Then, we compare them with those from models of Kavajecz (1998), Chen et al. (2000), and Dupont (2000). Finally, our numerical outcomes consistent with extant empirical results, and those being new and feasible in testing are illustrated.

Our empirically testable findings derived in Section 6 are listed below.

Result (i): The market maker could trade on one side of the market by quoting small depths.

Result (ii): The market maker would slightly raise spreads and lower

¹³ The original outcomes in Section 6 rely on the conditions of $\partial^2 \eta^{b*} / \partial \bar{S}^2 < \partial^2 X_0^{b*} / \partial \bar{S}^2$ for large \bar{S} and $\partial^2 \eta^{b*} / \partial (q^a)^2 > \partial^2 X_0^{b*} / \partial (q^a)^2$ by footnote 10. Here, the associated properties still hold since $\partial^2 \tilde{\eta}^{b*} / \partial \bar{S}(\beta^*)^2 < \partial^2 \eta^{b*} / \partial \bar{S}(\beta^*)^2 < \partial^2 X_0^{b*} / \partial \bar{S}^2 < \partial^2 \tilde{X}_0^{b*} / \partial \bar{S}(\beta^*)^2$, and $\partial^2 \tilde{\eta}^{b*} / \partial (q^a)^2 = \partial^2 \eta^{b*} / \partial (q^a)^2 > \partial^2 X_0^{b*} / \partial (q^a)^2 = \partial^2 \tilde{X}_0^{b*} / \partial (q^a)^2$, where $\partial^2 \tilde{\eta}^{b*} / \partial \bar{S}(\beta^*)^2 = \partial^2 \eta^{b*} / \partial \bar{S}(\beta^*)^2 - 2\beta^*(q^a + S) / [\bar{S}(\beta^*) - B(\beta^*) - q^a - S]^3$ and $\partial^2 \tilde{X}_0^{b*} / \partial \bar{S}(\beta^*)^2 = (1/2)(\gamma - \beta/2)^{1/2} (\delta \sigma_\eta^2)^{-1/2} [\bar{S}(\beta^*) - B(\beta^*)]^{-3/2}$.

depths when the ratio of informed traders increases.

Result (iii): For relatively small variance of informed traders' private information, both equilibrium spreads and depths increase significantly as the variance increases. However, for relatively large variance of informed traders' private information, only equilibrium spreads increase significantly with rising variance. The former implies that equilibrium spreads and depths move in the same direction, hence leaving the impact of changing variance of informed traders' private information on market maker's liquidity inconclusive.

Result (iv): Equilibrium spreads and depths will increase significantly and slightly respectively, when risk-averse levels of uninformed traders rise. In other words, equilibrium spreads and depths would move in the same direction. Thus, the impact of changing risk-averse levels of uninformed traders on market maker's liquidity is ambiguous.

Result (v): When magnitudes of liquidity shocks increase, the market maker will increase quoted depths and quoted spreads. Thus, equilibrium spreads and depths vary in the same direction.

Result (vi): When expected future market order flows increase, the market maker will narrow quoted spreads but increase quoted depths. Thus, equilibrium spreads and depths move in the opposite direction.

Result (vii): When limit-buy or limit-sell books become thicker and equilibria are in area A, the market maker would increase spreads and decrease depths. Nevertheless, the market maker would lower spreads slightly but reduce depths significantly as limit-buy or limit-sell books become thicker and equilibria occur in area E or F.

Since our model has the same setting of informed traders as those in Kavajecz (1998), Chen et al. (2000), and Dupont (2000), Result (ii) is also observed in these models. Nevertheless, our Result (iii) is merely found in Chen et al. (2000). On the other hand, uninformed traders are assumed to be risk-averse in Chen et al. (2000) and here, but presumed risk-neutral in Kavajecz (1998) and Dupont (2000). Thus, our Result (iv) is absent in Kavajecz (1998) and Dupont (2000), but observed in Chen et al. (2000). Because Kavajecz (1998) and Chen et al. (2000) also consider liquidity shocks of uninformed traders, our Result (v) appears in the two models. The major distinction between our model and the setups of Kavajecz (1998), Chen et al. (2000), and Dupont (2000) is that they do not allow traders to submit limit orders. Thus, Result (i), Result (vi), and Result (vii) appear in this study only.

When comparing our numerical outcomes with previous empirical results, we find that Result (i) is consistent with Kavajecz's (1999) and Result (iii) is consistent with Liu et al.'s (2001). Our Result (ii), Result (iv), and Result (v) may not be tested easily because ratios of informed traders, risk-averse levels of uninformed traders, and magnitudes of liquidity shocks are hard to measure. Nevertheless, Result (iv) and Result (vii) are our new findings and should be feasible in testing.

9. Conclusions

In this paper, a static quote-driven trading model with three kinds of strategic players is developed. The major innovation of our model is to endogenize the quoted-depth variable and, at the same time, to allow traders to choose between market and limit orders. We characterize how traders' optimal submission strategies are affected by a monopolistic market maker's quotations, limit order books, and expected future

market order flows. On the other hand, since theoretical closed-form solutions of our model are too difficult to be derived, market maker's equilibrium quotations are solved numerically. Comparative statics analyses are also conducted numerically. The outcomes show that the market maker may trade merely on one side of the market under specific conditions. Finally, both ambiguous (same-direction) and unambiguous (opposite-direction) movements of equilibrium spreads and depths are observed when trading-environment parameters vary. The former would result in inconclusive situations about the impact of changing parameters on market maker's liquidity provision.

Appendix

Proof of Lemma 1. There are five cases below.

Case (a): Suppose $\eta \geq \gamma > 0$, and $0 < (\bar{S} - B) < (q^a + S)$.

By (5) and (6), we have

$$\begin{aligned} E\pi^l(b_i^{ls} | \eta, \gamma, q^a) &\geq (\leq) E\pi^l(q^a + S | \eta, \gamma, q^a) \\ \Leftrightarrow (\bar{S} - B)(\eta + \gamma) &\geq (\leq) (q^a + S)(\eta - \gamma) \\ \Leftrightarrow \eta &\leq (\geq) \eta^{bs}. \end{aligned}$$

So, informed traders will submit market-buys if $\eta \geq \eta^*$, and limit-buys if $\eta \leq \eta^{bs}$. It is easy to see that $\eta^{bs} \geq \gamma$.

Case (b): Suppose $\eta \geq \gamma > 0$ and $(\bar{S} - B) > (q^a + S) > 0$.

Under the circumstance, we have $E\pi^l(b_i^{ls} | \eta, \gamma, q^a) > E\pi^l(q^a + S | \eta, \gamma, q^a)$ by (5)-(6). Thus, informed traders will submit limit-buys.

Case (c): Suppose $-\gamma < \eta < \gamma$, $(\bar{B} - S) > 0$, and $(\bar{S} - B) > 0$.

In this case, we have $E\pi^l(b_i^m | \eta, \gamma, q^a) < 0$ and $E\pi^l(b_i^l | \eta, \gamma, q^a) > 0$ for all $b_i^m, b_i^l > 0$. Also, $E\pi^l(s_i^m | \eta, \gamma, q^b) < 0$ and $E\pi^l(s_i^l | \eta, \gamma, q^b) > 0$ for all $s_i^l, s_i^m > 0$. Thus, informed traders would submit limit orders.

Case (d): Suppose $\eta \leq -\gamma < 0$ and $(q^b + B) > (\bar{B} - S) > 0$.

By (7) and (8), we have

$$\begin{aligned} E\pi^l(s_i^{ls} | \eta, \gamma, q^b) &\geq (\leq) E\pi^l(q^b + B | \eta, \gamma, q^b) \\ \Leftrightarrow (\bar{B} - S)(\gamma - \eta) &\geq (\leq) (q^b + B)(-\gamma - \eta) \\ \Leftrightarrow \eta &\geq (\leq) \eta^{ss}. \end{aligned}$$

Thus, if $\eta \leq \eta^{ss}$, informed traders will submit market-sells. Otherwise,

limit-sells should be submitted. It is easy to see $\eta^{*s} \leq -\gamma < 0$, hence $\eta^{*s} < \eta^{*b}$.

Case (e): Suppose $\eta \leq -\gamma < 0$ and $0 < (q^b + B) < (\bar{B} - S)$.

In this case, we have $E\pi^i(s_i^* | \eta, \gamma, q^b) > E\pi^i(q^b + B | \eta, \gamma, q^b)$. Therefore, informed traders would submit limit-sells.

Proof of Lemma 2. Suppose $X_0 < 0$ and uninformed traders submit a b^{*s} -unit limit-buy. Then, b^{*s} must solve the problem of

$$\begin{aligned} \max_{b^i} \quad & W_0 + b^i E(\tilde{F}^{b^i})\gamma - \left(\frac{\delta}{2}\right)[X_0 + b^i E(\tilde{F}^{b^i})]^2 \sigma_\eta^2, \\ \text{s.t.} \quad & b^i E(\tilde{F}^{b^i}) \leq (\bar{S} - B). \end{aligned}$$

Let $L \equiv W_0 + b^i E(\tilde{F}^{b^i})\gamma - (\delta/2)[X_0 + b^i E(\tilde{F}^{b^i})]^2 \sigma_\eta^2 - \lambda[b^i E(\tilde{F}^{b^i}) - (\bar{S} - B)]$. The first order conditions are

$$\begin{aligned} \frac{\partial L}{\partial b^i E(\tilde{F}^{b^i})} &= \gamma - \delta\sigma_\eta^2[X_0 + b^i E(\tilde{F}^{b^i})] - \lambda = 0, \\ \frac{\partial L}{\partial \lambda} &= b^i E(\tilde{F}^{b^i}) - (\bar{S} - B), \quad \frac{\partial L}{\partial \lambda} \cdot \lambda = 0, \quad \text{and} \quad \lambda \geq 0. \end{aligned}$$

So, $b^{*s} E(\tilde{F}^{b^{*s}}) = (\bar{S} - B)$ if $\lambda > 0$, and $b^i E(\tilde{F}^{b^i}) = \gamma/(\delta\sigma_\eta^2) - X_0$ if $\lambda = 0$. However, if $X_0 < \gamma/(\delta\sigma_\eta^2) - (\bar{S} - B)$, we have $\lambda = \gamma - \delta\sigma_\eta^2[X_0 + b^{*s} E(\tilde{F}^{b^{*s}})] > -\delta\sigma_\eta^2[b^{*s} E(\tilde{F}^{b^{*s}}) - \bar{S} + B] \geq 0$. It follows that $\lambda > 0$. For $X_0 \geq \gamma/(\delta\sigma_\eta^2) - (\bar{S} - B)$, we must have $\lambda = 0$. Suppose not. Then we have $b^{*s} E(\tilde{F}^{b^{*s}}) = (\bar{S} - B)$ and $\lambda = \gamma - \delta\sigma_\eta^2(X_0 + \bar{S} - B) \leq 0$. This is a contradiction. Thus, $\lambda = 0$ for $X_0 \geq \gamma/(\delta\sigma_\eta^2) - (\bar{S} - B)$. The proof for part (i) is completed, and the proof for part (ii) can be shown similarly.

Proof of Proposition 1. Only proof for $(q^a + S) > \gamma/(\delta\sigma_\eta^2) > (\bar{S} - B) > 0$ is provided here. The outcomes for both $\gamma/(\delta\sigma_\eta^2) > (q^a + S) > (\bar{S} - B) > 0$ and $(q^a + S) > (\bar{S} - B) > \gamma/(\delta\sigma_\eta^2)$ can be shown similarly.

Part (i): By (15) and Lemma 2(ii), uninformed traders' optimal trading quantities using market and limit orders are $b^{ls} E(\tilde{F}^{b^{ls}}) = (\bar{S} - B)$ for all $X_0 \in [-g, 0]$, and

$$b^{ms} = \begin{cases} 0 & \text{if } X_0 \in [\frac{-\gamma}{\delta\sigma_\eta^2}, 0], \\ -\frac{\gamma}{\delta\sigma_\eta^2} - X_0 & \text{if } X_0 \in [\frac{-\gamma}{\delta\sigma_\eta^2} - q^a - S, \frac{-\gamma}{\delta\sigma_\eta^2}], \\ q^a + S & \text{if } X_0 \in [-g, \frac{-\gamma}{\delta\sigma_\eta^2} - q^a - S], \end{cases}$$

respectively. By dividing interval $[-g, 0]$ into three disjoint subintervals, we can derive uninformed traders' optimal submission strategies as follows.

(a) If $X_0 \in [-\gamma/(\delta\sigma_\eta^2), 0]$, then $b^{ms} = 0$ and $b^{ls} E(\tilde{F}^{b^{ls}}) = (\bar{S} - B)$.

Therefore, by (14) and (19), we have

$$\begin{aligned} & U^m(0) - U^l(\bar{S} - B) \\ &= W_0 - \frac{\delta}{2} \sigma_\eta^2 X_0^2 - W_0 - (\bar{S} - B)\gamma + \frac{\delta}{2} [X_0 + (\bar{S} - B)]^2 \sigma_\eta^2 \\ &= (\bar{S} - B)[- \gamma + \delta\sigma_\eta^2 X_0 + \frac{\delta\sigma_\eta^2}{2} (\bar{S} - B)] < 0, \end{aligned}$$

due to $\gamma/(\delta\sigma_\eta^2) > \bar{S} - B$ and $X_0 < 0$. Thus, it is optimal for uninformed traders to submit limit-buys.

(b) If $X_0 \in [-\gamma/(\delta\sigma_\eta^2) - q^a - S, -\gamma/(\delta\sigma_\eta^2)]$, then $b^{ms} = -\gamma/(\delta\sigma_\eta^2) - X_0$ and $b^{ls} E(\tilde{F}^{b^{ls}}) = (\bar{S} - B)$. Thus, we have

$$\begin{aligned}
& U^m\left(\frac{-\gamma}{\delta\sigma_\eta^2} - X_0\right) - U^l(\bar{S} - B) \\
&= -\gamma\left[\frac{-\gamma}{\delta\sigma_\eta^2} - X_0 + \bar{S} - B\right] \\
&\quad - \frac{\delta\sigma_\eta^2}{2}\left[\frac{-\gamma}{\delta\sigma_\eta^2} - X_0 - (\bar{S} - B)\right]\left[\frac{-\gamma}{\delta\sigma_\eta^2} + X_0 + \bar{S} - B\right]. \tag{A1}
\end{aligned}$$

Here $U^m(-\gamma/(\delta\sigma_\eta^2) - X_0) - U^l(\bar{S} - B) \leq 0$, when $-\gamma/(\delta\sigma_\eta^2) - X_0 \leq (\bar{S} - B)$, because $(\bar{S} - B) < \gamma/(\delta\sigma_\eta^2)$ and $X_0 < 0$. Now suppose $-\gamma/(\delta\sigma_\eta^2) - X_0 \geq (\bar{S} - B)$. By rearranging (A1), we get

$$\begin{aligned}
& U^m\left(\frac{-\gamma}{\delta\sigma_\eta^2} - X_0\right) - U^l(\bar{S} - B) \\
&= \left(\frac{\delta\sigma_\eta^2}{2}\right)X_0^2 + X_0[\gamma + \delta\sigma_\eta^2(\bar{S} - B)] \\
&\quad + \left[\frac{\gamma^2}{2\delta\sigma_\eta^2} - \gamma(\bar{S} - B) + \left(\frac{\delta\sigma_\eta^2}{2}\right)(\bar{S} - B)^2\right]. \tag{A2}
\end{aligned}$$

When $q^a + S \geq \bar{S} - B + 2\sqrt{\gamma(\bar{S} - B)/(\delta\sigma_\eta^2)}$, $U^m(-\gamma/(\delta\sigma_\eta^2) - X_0) \geq U^l(\bar{S} - B)$

for $X_0 \leq X_0^{b^*}$ and $U^m(-\gamma/(\delta\sigma_\eta^2) - X_0) \leq U^l(\bar{S} - B)$ for $X_0 \geq X_0^{b^*}$.¹⁴

¹⁴ Define $f_1^b(X_0) = U^m(-\gamma/(\delta\sigma_\eta^2) - X_0) - U^l(\bar{S} - B)$. The two roots of equation $f_1^b(X_0) = 0$ are $X_0^{b^*} = \left\{-[\gamma + \delta\sigma_\eta^2(\bar{S} - B)] - 2\sqrt{\gamma\delta\sigma_\eta^2(\bar{S} - B)}\right\}/(\delta\sigma_\eta^2)$ and $\hat{X}_0^{b^*} = \left\{-[\gamma + \delta\sigma_\eta^2(\bar{S} - B)] + 2\sqrt{\gamma\delta\sigma_\eta^2(\bar{S} - B)}\right\}/(\delta\sigma_\eta^2)$. Since $\hat{X}_0^{b^*} > -\gamma/(\delta\sigma_\eta^2) - (\bar{S} - B)$ contradicting the hypothesis of $-\gamma/(\delta\sigma_\eta^2) - X_0 \geq \bar{S} - B$, it remains to check whether $X_0^{b^*} \in [-\gamma/(\delta\sigma_\eta^2) - (q^a + S), -\gamma/(\delta\sigma_\eta^2)]$. Obviously, $X_0^{b^*} \leq -\gamma/(\delta\sigma_\eta^2)$. On the other hand, $X_0^{b^*} - [-\gamma/(\delta\sigma_\eta^2) - q^a - S] = q^a + S - (\bar{S} - B) - 2\sqrt{\gamma(\bar{S} - B)/(\delta\sigma_\eta^2)}$. Thus, $X_0^{b^*} \in [-\gamma/(\delta\sigma_\eta^2) - (q^a + S), -\gamma/(\delta\sigma_\eta^2)]$ if and only if $q^a + S \geq \bar{S} - B + 2\sqrt{\gamma(\bar{S} - B)/(\delta\sigma_\eta^2)}$. In addition, it is easy to check that $f_1^b[-\gamma/(\delta\sigma_\eta^2) - (\bar{S} - B)] = -2(\bar{S} - B)\gamma < f_1^b(X_0^{b^*}) = 0$. Hence, $f_1^b(X_0) \leq 0$ as $X_0 \geq X_0^{b^*}$ and $f_1^b(X_0) \geq 0$ as $X_0 \leq X_0^{b^*}$.

This means that liquidity traders prefer market-buys when $X_0 \leq X_0^{b^*}$, and prefer limit-buys when $q^a + S \geq \bar{S} - B + 2\sqrt{\gamma(\bar{S} - B)/(\delta\sigma_\eta^2)}$. Additionally, if $q^a + S < \bar{S} - B + 2\sqrt{\gamma(\bar{S} - B)/(\delta\sigma_\eta^2)}$, then $X_0^{b^*} < -\gamma/(\delta\sigma_\eta^2) - (q^a + S)$. Hence, $U^m(-\gamma/(\delta\sigma_\eta^2) - X_0) \leq U^l(\bar{S} - B)$ for all $X_0 \in [-\gamma/(\delta\sigma_\eta^2) - (q^a + S), -\gamma/(\delta\sigma_\eta^2)]$. This means that limit orders should be chosen by uninformed traders in this case.

(c) If $X_0 \in [-g, -\gamma/(\delta\sigma_\eta^2) - q^a - S]$, then $b^{m^*} = (q^a + S)$ and $b^{l^*} E(\tilde{F}^{b^{l^*}}) = (\bar{S} - B)$. Thus,

$$\begin{aligned} & U^m(q^a + S) - U^l(\bar{S} - B) \\ &= -\gamma[q^a + S + \bar{S} - B] + \frac{\delta\sigma_\eta^2}{2}[X_0 + \bar{S} - B + X_0 + q^a + S][\bar{S} - B - q^a - S]. \end{aligned} \quad (A3)$$

Under the condition of $q^a + S \leq \bar{S} - B + 2\sqrt{\gamma(\bar{S} - B)/(\delta\sigma_\eta^2)}$, we have $U^m(q^a + S) - U^l(\bar{S} - B) \geq 0$ for $X_0 \leq X_0^{b^{m^*}} < 0$, and $U^m(q^a + S) - U^l(\bar{S} - B) \leq 0$ for $0 > X_0 \geq X_0^{b^{m^*}}$.¹⁵

¹⁵ Define $f_2^b(X_0) = U^m(q^a + S) - U^l(\bar{S} - B)$. It is easy to check that $f_2^b(X_0^{b^{m^*}}) = 0$. Hence, $f_2^b(X_0) \geq 0$ if $X_0 \leq X_0^{b^{m^*}} < 0$, and $f_2^b(X_0) \leq 0$ if $0 > X_0 \geq X_0^{b^{m^*}}$. It remains to check whether $X_0^{b^{m^*}} \in [-g, -\gamma/(\delta\sigma_\eta^2) - q^a - S]$. Since $(\bar{S} - B) < (q^a + S)$ is assumed, equation $[X_0^{b^{m^*}} + \gamma/(\delta\sigma_\eta^2) + q^a + S] = [4\gamma(\bar{S} - B) - \delta\sigma_\eta^2(\bar{S} - B - q^a - S)^2]/[2\delta\sigma_\eta^2(\bar{S} - B - q^a - S)]$ has a negative denominator. Therefore, $[X_0^{b^{m^*}} + \gamma/(\delta\sigma_\eta^2) + q^a + S] \geq (\leq) 0$ if $q^a + S \geq (\leq) \bar{S} - B + 2\sqrt{\gamma(\bar{S} - B)/(\delta\sigma_\eta^2)}$. It means that if $q^a + S \geq \bar{S} - B + 2\sqrt{\gamma(\bar{S} - B)/(\delta\sigma_\eta^2)}$, $X_0^{b^{m^*}}$ exists but $X_0^{b^{m^*}} \notin [-g, -\gamma/(\delta\sigma_\eta^2) - q^a - S]$. Then, $U^m(q^a + S) - U^l(\bar{S} - B) \geq 0$ for all $X_0 \in [-g, -\gamma/(\delta\sigma_\eta^2) - q^a - S]$. The proof for Proposition 1(i) is thus completed. Conversely, if $q^a + S \leq \bar{S} - B + 2\sqrt{\gamma(\bar{S} - B)/(\delta\sigma_\eta^2)}$, then $X_0^{b^{m^*}}$ exists but $X_0^{b^{m^*}}$ does not. Therefore, we get $U^m(q^a + S) - U^l(\bar{S} - B) \geq 0$ for $X_0 \leq X_0^{b^{m^*}} < 0$ and $U^m(q^a + S) - U^l(\bar{S} - B) \leq 0$ for $X_0 \geq X_0^{b^{m^*}}$. The proof for Proposition 1(i) is now done.

Part (ii). Again, only proof for $\gamma/(\delta\sigma_\eta^2) > (\bar{S} - B) > (q^a + S)$ is provided here. Uninformed traders' optimal trading quantities using market and limit orders are the same as those in part (i). Therefore, we consider the ensuing three cases.

- (a) For $X_0 \in [-\gamma/(\delta\sigma_\eta^2), 0]$, the proof is the same as that of part (ia).
 (b) For $X_0 \in [-\gamma/(\delta\sigma_\eta^2) - q^a - S, -\gamma/(\delta\sigma_\eta^2)]$, we have $U^m(-\gamma/(\delta\sigma_\eta^2) - X_0) - U^l(\bar{S} - B) \leq 0$ since $0 \leq -\gamma/(\delta\sigma_\eta^2) - X_0 \leq q^a + S < \bar{S} - B$, $\gamma/(\delta\sigma_\eta^2) > \bar{S} - B$ and $X_0 < 0$.
 (c) For $X_0 \in [-g, -\gamma/(\delta\sigma_\eta^2) - q^a - S]$, we have $U^m(q^a + S) - U^l(\bar{S} - B) = -\gamma[q^a + S + \bar{S} - B] + (\delta\sigma_\eta^2/2)[2X_0 + \bar{S} - B + q^a + S][\bar{S} - B - q^a - S] \leq 0$ since $X_0 \leq -\gamma/(\delta\sigma_\eta^2) \leq -(\bar{S} - B) \leq -(q^a + S) \leq 0$.

Thus, from cases (a)-(c), it is proved that limit-buys are uninformed traders' optimal choices for all $X_0 \in [-g, 0]$.

Uninformed traders' optimal trading quantities using limit-sells

Lemma 2A. (i) Suppose $(\bar{B} - S) > \gamma/(\delta\sigma_\eta^2)$, Then uninformed traders' optimal trading quantities for limit-sells, s^{ls} , satisfy

$$s^{ls} E(\tilde{F}^{s^{ls}}) = \begin{cases} \frac{\gamma}{\delta\sigma_\eta^2} + X_0 & \text{if } X_0 \in [0, \frac{-\gamma}{\delta\sigma_\eta^2} + (\bar{B} - S)], \\ (\bar{B} - S) & \text{if } X_0 \in [\frac{-\gamma}{\delta\sigma_\eta^2} + (\bar{B} - S), g]. \end{cases}$$

(ii) Suppose $(\bar{B} - S) < \gamma/(\delta\sigma_\eta^2)$, Then uninformed traders' optimal trading quantities for limit-sells, s^{ls} , satisfy $s^{ls} E(\tilde{F}^{s^{ls}}) = (\bar{B} - S)$ for all $X_0 \in [0, g]$.

Uninformed traders' optimal submission strategies on the sell side

Proposition 2. (i) Suppose $(q^b + B) > \gamma/(\delta\sigma_\eta^2) > (\bar{B} - S) > 0$, $\gamma/(\delta\sigma_\eta^2) > (q^b + B) > (\bar{B} - S)$, or $(q^b + B) > (\bar{B} - S) > \gamma/(\delta\sigma_\eta^2)$. For $X_0 \in [0, g]$, we have the followings.

(ia) If $q^b + B \geq \bar{B} - S + 2\sqrt{\gamma(\bar{B} - S)/(\delta\sigma_\eta^2)}$, then submitting limit-sells is optimal for $X_0 \leq X_0^{s*}$, and submitting market-sells is optimal for $X_0 \geq X_0^{s*}$, where $X_0^{s*} \equiv \{[\gamma + \delta\sigma_\eta^2(\bar{B} - S)] + 2\sqrt{\gamma\delta\sigma_\eta^2(\bar{B} - S)}\}/(\delta\sigma_\eta^2)$.

(ib) If $q^b + B \leq \bar{B} - S + 2\sqrt{\gamma(\bar{B} - S)/(\delta\sigma_\eta^2)}$, then submitting limit-sells is optimal for $X_0 \leq X_0^{s**}$, and submitting market-sells is optimal for $X_0 \geq X_0^{s**}$, where $X_0^{s**} \equiv \gamma(q^b + B + \bar{B} - S)/[\delta\sigma_\eta^2(q^b + B - \bar{B} + S)] + (q^b + B - \bar{B} + S)/2$.

(ii) Suppose $\gamma/(\delta\sigma_\eta^2) > (\bar{B} - S) > (q^b + B)$, $(\bar{B} - S) > (q^b + B) > \gamma/(\delta\sigma_\eta^2)$, or $(\bar{B} - S) > \gamma/(\delta\sigma_\eta^2) > (q^b + B)$. Then uninformed traders prefer limit-sells to market-sells for $X_0 \in [0, g]$.

Comparative statics results of uninformed traders on the sell side of the market

Corollary 2A. (i) With rising quoted spreads, uninformed traders are more likely to submit limit-sells. That is,

$$\partial X_0^{s**} / \partial \gamma = 1/(\delta\sigma_\eta^2) + [(\bar{B} - S)/\gamma(\delta\sigma_\eta^2)]^{1/2} > 0 \text{ and}$$

$$\partial X_0^{s**} / \partial \gamma = (q^b + B + \bar{B} - S)/[\delta\sigma_\eta^2(q^b + B - \bar{B} + S)] > 0.$$

(ii) With rising quoted depths, uninformed traders are more likely to submit market-sells. That is, $\partial X_0^{s**} / \partial q^b = -2\gamma(\bar{B} - S)/[\delta\sigma_\eta^2(q^b + B - \bar{B} + S)^2] + 1/2 < 0$.

(iii) With increasing unexecuted limit-sells, uninformed traders are more

likely to place market-sells. That is,

$$\partial X_0^{s**} / \partial S = -1 - \{\gamma / [(\bar{B} - S)\delta\sigma_\eta^2]\}^{1/2} < 0 \quad \text{and}$$

$$\partial X_0^{s**} / \partial S = -2\gamma(q^b + B) / [\delta\sigma_\eta^2(q^b + B - \bar{B} + S)^2] - 1/2 < 0.$$

(iv) *With increasing unexecuted limit-buys, uninformed traders are more likely to place market-sells. That is, $\partial X_0^{s**} / \partial B = -2\gamma(\bar{B} - S) / [\delta\sigma_\eta^2(q^b + B - \bar{B} + S)^2] + 1/2 < 0$.*

(v) *With rising expected future market order flows, uninformed traders are more likely to place limit-sells. That is,*

$$\partial X_0^{s**} / \partial \bar{S} = 2\gamma(q^b + B) / [\delta\sigma_\eta^2(q^b + B - \bar{B} + S)^2] + 1/2 > 0. \quad \text{and}$$

$$\partial X_0^{s**} / \partial \bar{B} = 1 + \{\gamma / [(\bar{B} - S)\delta\sigma_\eta^2]\}^{1/2} > 0.$$

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價差、深度、及市價單和限價單 的最適下單策略

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摘 要

本文分析一個獨占的造市者如何決定其最適的報價價差和深度，以及資訊交易者和非資訊交易者的最適下單策略。我們發現報價深度對交易者的最適下單決策有顯著的影響。因此，造市者可透過調整報價深度以避免來自市場上買、賣一方或雙方的不利交易。此外，我們亦證明當價差變小、深度增加、同邊的限價單數量增加、或預期未來的市價單減少時，交易者較願意採用市價單。相反的情況下，交易者則較願意採用限價單。

關鍵詞：買賣價差、報價深度、市價單、限價單

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