Announcement Effects of Consumption Taxes

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Abstract

This paper investigates the effect of an increase in consumption taxes both in the steady state and along the transitional dynamics. We find that a higher consumption tax always reduces the household's welfare if we only investigate the steady state. If the government implements the policy without declaring it in advance, then a higher consumption tax also has a welfare cost. In addition, if we consider an announcement effect for a change in the consumption tax, then the welfare cost drops when the intertemporal elasticity of substitution for consumption is lower, but rises under a longer period of time between the policy's announcement and its implementation. The above results still hold under no leisure-labor trade-off, under an endogenous growth model, or under different utility functions.

Keywords: Announcement Effect, Consumption Tax, Welfare Cost
JEL Classification: E13, E62, H20

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1. Introduction

Public finances provide a critical way through which government expenditure and taxation policies affect private resource allocation and long-run welfare. Public economists have devoted considerable efforts to measuring the welfare of alternative ways of financing government spending. The existing literature has usually obtained that distortions caused by the consumption tax are less significant than those of other taxes. Thus, when the government needs to finance the extra public expenditure, an increase in the consumption tax could be a priority policy. In addition, there has been a long history of proposals for fundamental tax reform to replace the current income taxes with a consumption tax.[1]

When the government executes such a policy, it usually proclaims that policy in advance. Once a rate change is announced, a utility-maximizing household and a profit-maximizing firm will change their behavior, which induces an impact on the economy even before the policy is implemented. Thus, in this paper, we study the effect of an increase in consumption taxes both in the steady state and along the transitional dynamics. In the latter case, we consider two situations. One is that the government executes the policy without making an announcement in advance, while the other is that where the government announces the policy before implementing it, thereby giving rise to an influence called the announcement effect.

The main results of this paper are as follows. First, if we only investigate the steady state, a higher consumption tax always lowers the household’s welfare. Intuitively, a higher tax rate on consumption increases the price of consumption relative to leisure. The household will replace consumption with leisure, and so long-run labor and consumption are reduced. Due to the complementarity between labor and capital in the production function, capital is reduced as well, as is the production of output. Although higher leisure increases the household’s utility, lower consumption decreases it. As output is ultimately reduced, a higher consumption tax always results in a welfare cost.

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1 Examples can be found in Summers (1981), Auerbach et al. (1983), Gravelle (1991), and so forth.

Moreover, we also consider the impact of an increase in the tax rate on consumption along the transitional dynamics. Higher leisure increases the household's utility in the short run, but an increase in the consumption tax rate eventually hurts the long-run capital, labor, output and consumption. Thus a higher consumption tax still has a welfare cost if the government implements the policy without declaring it in advance. The formal analysis is supplemented by quantitative results, and the above conclusion holds under different levels of the intertemporal elasticity of substitution (hereafter, IES) for consumption.

Finally, if we are concerned about an announcement effect of a change in the consumption tax, the welfare cost is reduced when the IES for consumption is lower, but is increased under the longer period between the time of the policy's announcement and that of its implementation. Intuitively, the households are more willing to engage in intertemporal substitution under a higher IES for consumption. In other words, the level of intertemporal substitution is smaller under a lower IES for consumption. The fluctuations in capital, labor, output and consumption are smaller; in particular, the reductions in those variables are smaller under a lower IES for consumption. Thus the utility (welfare) is higher under that case.

In addition, the simulation results show that if the government declares the policy much earlier, the period in which the short-run utility (consumption) is higher than its initial level is longer, while the longer-term utility is lower due to the lower capital and output. If the former effect dominates, the welfare cost of the announced consumption tax is smaller when the duration between the time of the policy's announcement and that of its implementation increases, but that welfare cost is higher if the latter effect dominates. From the numerical exercises it can be inferred that the welfare cost of the announced consumption tax is larger when the government declares the policy much earlier.

Regarding the actual data in reality, Fuse (2004) estimated the IES for consumption in Japan and found that it was around 4, which is higher than in other countries. For example, the estimate of the IES for consumption in the United States, which is around 0.4, can be found in Ogaki and Reinhart

(1998).[2] Patterson and Pesaran (1992) estimated the IES for consumption in the United States and the United Kingdom and obtained a value for the US that was smaller than that for the UK, with both values being less than 0.5 in the two countries. In addition, Reinhart and Végh (1995) also surveyed the IES for consumption, which was usually less than one, in Latin and South American countries.

In this paper, we obtain that a higher consumption tax reduces the output produced and the household's welfare when the IES for consumption is high. By comparing our model with actual situations, an increase in consumption tax in Japan may result in a larger welfare cost than in the US. In addition, if the government of Japan had announced the policy earlier, the situation would be worse, i.e., the welfare cost would be higher. Furthermore, our results still hold when there is no leisure-labor trade-off, as well as under the endogenous growth model, and under different utility functions.

The contribution of this paper is that we can provide a suggestion to the government. When the government plans to implement certain policies, especially to enhance the tax rates, it should not announce the policies too early, particularly when the IES for consumption in the country is high. However, if the government wants to enhance the short-term consumption, announcing the policies much earlier may be a good approach to implement. Our simulation results are consistent with the actual data in Japan. During the period between the time of the policy's announcement and that of its implementation, consumption increases, while consumption declines after the policy is implemented.

Related papers like Garner (2005), which analyzed the macroeconomic effects of replacing the federal tax system with a consumption tax, have found that the key difference between an income tax and a consumption tax lies in the treatment of saving, and thus each tax provides different incentives to save and invest. Moreover, Lu et al. (2011) investigated welfare costs between seignorage and consumption taxes in a Neoclassical

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2 The estimate of the IES for consumption in the US can also be found in Hall (1988), Beaudry and Wincoop (1996), and so on.

growth model, and found that the welfare cost of the latter was larger than the welfare cost of the former along transitional dynamic and steady-state paths. However, neither Garner (2005) nor Lu et al. (2011) discussed the announcement effects of implemented taxes. Since the government usually proclaims its policy before executing it, this paper can overcome the shortcomings of related existing studies that ignore the lapse of time between an announcement and its implementation.

The structure of the remainder of this paper is as follows. In Section 2, we set up a one-sector neoclassical growth model and analyze the individual's optimizations. Section 3 studies the equilibrium and the comparative static analysis. We then use numerical simulation to discuss the transitional dynamics and the announcement effects. Section 4 provides some robustness checks. Concluding remarks are offered in Section 5. Finally, technical details are relegated to the Appendix.

2. The Benchmark Model

We consider a continuous-time Ramsey model. The economy is populated by a continuum of identical infinitely-lived households (of measure one), a continuum of identical firms (of measure one), and a fiscal authority.

2.1 Households

The representative household has a unit of time endowment of which a fraction *l* of the time endowment is allocated to work and the remaining fraction is allocated to leisure. The household decides its working time, consumption (*c*) and savings at each point in time. The lifetime welfare of the representative household is represented by

$$U = \int_{t=0}^{\infty} u(c,l) e^{-\rho t} dt, \tag{1}$$

where $\rho > 0$ is the time preference rate, $u_c(c,l) > 0 > u_{cc}(c,l)$ and $u_l(c,l) < 0$ since higher working implies lower leisure. To simplify the analysis, we first use a separable utility function between consumption and labor, i.e., $u_{cl}(c,l)=0$, and later discuss the results under other utility functions.

Denote k as capital with δ as its depreciation rate. Furthermore, denote w and r as the wage rate and the rental rate, respectively. At any point in time, the representative household's budget constraint is

$$\dot{k} = wl + rk - \delta k - (1+\tau_c)c + T, \tag{2}$$

where τ_c is the tax rate on consumption and T represents lump-sum transfers from the government.

The representative household's problem is to maximize the lifetime preference in (1) by choosing between consumption, labor, and investment, subject to the budget constraint (2), taking as given the tax rates, transfers, factor prices, and the given initial capital $k(0)$. Denote λ as the Lagrange multiplier on constraint (2). The necessary conditions are:

$$u_c(c,l) = \lambda(1+\tau_c), \tag{3a}$$

$$-u_l(c,l) = \lambda w, \tag{3b}$$

$$-\lambda(r-\delta) = \dot{\lambda} - \rho\lambda, \tag{3c}$$

along with the transversality condition,

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)k(t) = 0. \tag{3d}$$

The conditions above are standard: (3a) determines optimal consumption, while (3b) denotes the tradeoff between labor supply and leisure, (3c) is the Euler equation, and (3d) is the usual transversality or "no Ponzi game" condition on capital.

To simplify these conditions, (3a) and (3c) together yield the consumption Euler equation,

$$\dot{c} = -\frac{u_c(c,l)}{u_{cc}(c,l)}(r - \delta - \rho). \tag{4a}$$

In addition, (3a) and (3b) jointly produce the consumption-leisure trade-off condition as follows:

$$-\frac{u_c(c,l)}{u_l(c,l)} = \frac{1+\tau_c}{w}, \tag{4b}$$

which states that, in the optimum, the marginal rate of substitution between leisure and consumption is equal to the post-tax wage rate.

2.2 Firms

The representative firm produces a single final good y_t by renting capital and employing labor under the following Neoclassical production technology:

$$y = f(k,l), \tag{5}$$

where $f_k(k,l) > 0 > f_{kk}(k,l)$ and $f_l(k,l) > 0 > f_{ll}(k,l)$.

Taking factor prices as given, the representative firm chooses capital and labor in order to maximize the following profit:

$$\pi = f(k,l) - wl - rk. \tag{6}$$

The optimal conditions are:

$$r = f_k(k,l), \tag{7a}$$

$$w = f_l(k,l). \tag{7b}$$

The above conditions show that, in equilibrium, the marginal product of capital is equal to the rental rate, and the marginal product of labor is equal to the wage rate, respectively.

2.3 The Government

At a given point in time, the government receives consumption taxes. The government uses this tax revenue to finance a direct lump-sum transfer T under a balanced budget as follows:

$$T = \tau_c c. \tag{8}$$

It is worth noting that the transfer is included to ensure that the government budget is balanced in the presence of pre-existing taxes that fit the data observations. To simplify the model, we assume that the government has no other public expenditure. As in the line of research in the public finance approach, we abstract the effects of government expenditures and assume that government expenditures affect neither production nor preferences. This assumption isolates the distortions generated by government expenditures.

2.4 Equilibrium Conditions

In equilibrium, all markets must clear. As the government transfers tax revenues to the representative agent, using the household's budget constraint, (2), the firm's optimal conditions, (7a) - (7b), and the government's balanced budget constraint, (8), we obtain the following goods market clearance condition:

$$\dot{k} = f(k,l) - \delta k - c. \tag{9}$$

A perfect-foresight competitive equilibrium defines the time paths of the quantities $\{c,k,l\}$ and prices $\{r,w\}$ that satisfy (4a)-(4b), (7a)-(7b), and (9).

To determine the equilibrium, first, we substitute the firm's optimal condition of capital in (7a), and the consumption Euler equation in (4a) becomes:

$$\dot{c} = -\frac{u_c(c,l)}{u_{cc}(c,l)}[f_k(k,l) - \delta - \rho]. \tag{10}$$

Next, by using the firm's optimal condition of labor in (7b), the household's consumption-leisure tradeoff condition in (4b) becomes:

$$-\frac{u_c(c,l)}{u_l(c,l)} = \frac{1+\tau_c}{f_l(k,l)}. \tag{11}$$

Thus, given τ_c, we can use (9), (10) and (11) to determine the time paths of c, k and l.

3. Announcement Effects of Consumption Taxes

This section analyzes the effects of consumption taxes. We first study the long-run effects of a higher consumption tax rate on consumption, labor, capital, output and welfare in the steady state. Then, we calibrate the model and discuss the influence of a change in the consumption tax rate along the transitional dynamics.

3.1 The Steady State

In order to accurately analyze the effect of a consumption tax, we use explicit functions. Assume that the firm's production function follows the Cobb-Douglas production technology as follows:

$$y = f(k,l) = Al^{\alpha}k^{1-\alpha}, \tag{5'}$$

where $A > 0$ is productivity and $\alpha \in (0,1)$. In addition, the lifetime welfare of the representative household is represented by

$$U = \int_{t=0}^{\infty} u(c,l)e^{-\rho t}dt = \int_{t=0}^{\infty}\left(\frac{c^{1-\sigma}-1}{1-\sigma} - \chi\frac{l^{1+\varepsilon}}{1+\varepsilon}\right)e^{-\rho t}dt, \tag{1'}$$

where $\chi > 0$ measures the degree of disutility of working relative to consumption in utility. We use a conventional additively separable utility function between consumption and labor with an IES for consumption $(1/\sigma > 0)$, which is different from the labor elasticity $(1/\varepsilon > 0)$.

In the steady state, we have $\dot{k} = \dot{c} = 0,$ and thus *c*, *k* and *l* are constant. (9) and (10) can be expressed as the following equations, respectively.[3]

$$l^* = \left[\frac{\rho+\delta}{(1-\alpha)A}\right]^{\frac{1}{\alpha}} k^*, \tag{12a}$$

$$c^* = \left[\frac{\rho+\delta}{(1-\alpha)} - \delta\right] k^*. \tag{12b}$$

Substituting (12a) and (12b) into (11) gives

$$k^* = \left\{ \left[\frac{\rho+\delta}{(1-\alpha)} - \delta\right]^{-\sigma} \frac{\alpha A}{\chi(1+\tau_c)} \left[\frac{(1-\alpha)A}{\rho+\delta}\right]^{\frac{1-\alpha+\varepsilon}{\alpha}} \right\}^{\frac{1}{\alpha+\varepsilon}}. \tag{12c}$$

We find that an increase in the consumption tax decreases the steady-state capital stock, and thus consumption, labor, and output also decline in the long run. Intuitively, a higher tax rate on consumption increases the price of consumption relative to leisure. The household will replace consumption by leisure, so that in the long run labor and consumption are reduced. Due to the complementarity between labor and capital in the production function, capital is reduced as well, as is the production of output.

However, the household's welfare is not necessarily reduced. In the long run, we can use (1') to calculate the lifetime welfare as $U^* = \{[(c^*)^{1-\sigma} - 1/(1-\sigma)] - [\chi(l^*)^{1+\varepsilon}/(1+\varepsilon)]\}/\rho.$ The household may feel happier under a higher tax rate on consumption if the effect of lower labor (higher leisure) dominates that of lower consumption.

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3 The variables with the superscript* represent their steady-state values.

3.2 Calibration

To quantify the results, we calibrate the model in the steady state in order to reproduce key features that are representative of the Japan economy in quarterly frequencies. Kydland and Prescott (1991) used 4% as the annual rate of time preference. We assume that people in the developed countries have the same time preference, and thus $\rho = 1\%$. In the existing literature, it is more common to use a capital share of output of between 0.30 and 0.40 (see the examples in Cooley, 1995). We thus set the capital share of output at 0.36. Thus $\alpha = 1 - 0.36 = 0.64$. Besides, we set the initial tax rate on consumption at $\tau_c = 0.05$.

By setting the initial $c/y = 0.67$, we can combine (12a) with (12b) to calibrate the depreciation rate of capital as $\delta = 0.1100$. According the regulations of the Ministry of Health, Labour and Welfare in Japan, legal working hours are 40 hours per week (or 44 hours per week for workplaces which special measures are applied to), i.e., the fraction of productive time is around 0.238 (or 0.262).[4] We use a middle value and thus set l = 0.25. In addition, we normalize productivity, and thus $A = 1$. Using (12a), (12b), and the production function (5'), we can calibrate the initial values of consumption, capital and output as c = 0.3107, k = 1.3914, y = 0.4638, respectively.

Moreover, the IES for consumption in Japan is around 4 according to Fuse (2004), i.e., $\sigma = 0.25$. We thus set the IES for consumption at 4 as our benchmark case, i.e., $\sigma = 0.25$, but we will change the value of σ later to discuss the intertemporal substitution effects of consumption taxes. The labor elasticity ranges from close to 0 (MaCurdy, 1981) to 3.8 (Imai and Keane, 2004). We choose a middle value of the labor elasticity of 1.9 to represent the situation in Japan, which implies that $\varepsilon = 0.5263$. Furthermore, the degree of disutility of working relative to consumption in utility can be simulated from (12c). We obtain $\chi = 3.1416$. Finally, we can use the data and the above calibration values in the long run to calibrate the lifetime welfare as $U = -102.6473$.

4 As for the regulations of working hours in Japan, please refer to http://www.mhlw.go.jp/.

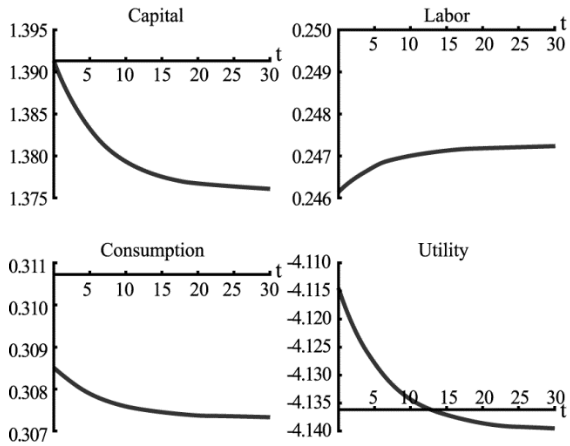
Now we can discuss the effects of a higher tax rate on consumption in the steady state. If τ_c is increased from 5% to 8%, we obtain that $c^* = 0.2997$, $l^* = 0.2411$, $k^* = 1.3418$, $y^* = 0.4473$, and $U^* = -102.8007$. The outcome is the same as in the analysis of our theoretical results, in which consumption, labor, capital and output are all reduced. In addition, the lifetime welfare is also decreased since the effect of less labor is dominated by the effect of lower consumption. Our numerical exercise shows that an increase in the tax rate on consumption not only reduces output, but also lowers the household's welfare in the steady state.

In addition, we also check the results for the US economy. As was mentioned in the Introduction, the IES for consumption in the United States is less than 1. We thus set the IES for consumption at 0.5, i.e., $\sigma = 2$. By following the same steps and setting, with the exception that $\sigma = 2$, we can calibrate the above macroeconomic variables in the US economy. The values of the initial capital, consumption and output are the same as those in the calibrated Japan economy, and initial lifetime welfare is $U = -413.6257$. When τ_c is increased from 5% to 8%, we obtain that $c^* = 0.3073$, $l^* = 0.2472$, $k^* = 1.3759$, $y^* = 0.4586$, and $U^* = -413.9974$. No matter what the initial values of σ are, the simulation results are similar. The higher consumption tax rate always reduces output and the lifetime welfare, and the situation becomes worse when the IES for consumption in that economy is larger.

3.3 Dynamic Analysis and Time Paths

The above section only investigates the long-run effect of consumption taxes. However, when the government executes the policy, the economy needs time to adjust and can not converge to its steady state immediately. In addition, the government usually announces its policy prior to bringing the policy into force. In this section, we first discuss the case where the government executes the policy without making an announcement in advance. Then, in the next section, we will analyze the case that usually applies in reality in which the government announces the policy before implementing it.

Assuming that the government increases the consumption tax rate from 5% to 8% at time 0, all agents will change their behavior in accordance with the new tax rate on consumption. According to the analysis in the above section, we know that output production and the household's welfare in the steady state are reduced under a higher tax rate on consumption. However, the households and the firms can not adjust to the long-run steady state immediately. If the output or utility in the short run is higher than its initial value, a higher consumption tax may favor the household's discounted sum of utility, i.e., the actual lifetime welfare.[5] That is, we need to investigate the transitional dynamics.[6] The time paths of related variables in accordance with a higher tax rate for consumption under $\sigma = 2$ and $\sigma = 0.25$ are depicted in Figures 1 and 2, respectively.

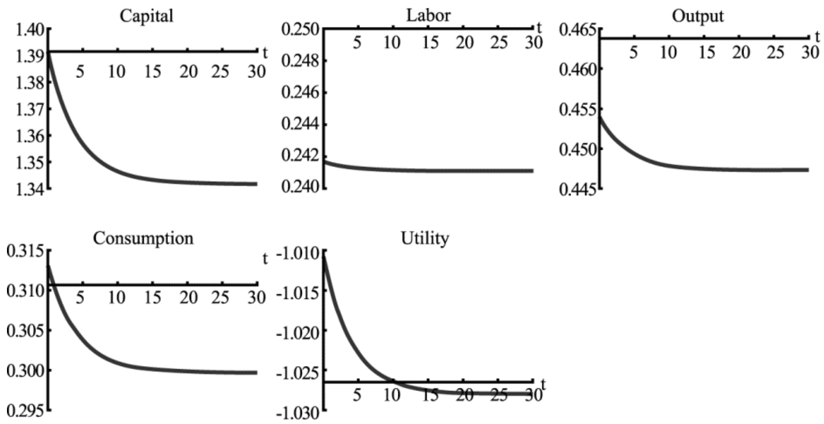


Note: The intersection of the horizontal and the vertical axes is the initial steady-state value under $\tau_c = 5\%$, and the levels of utility are calculated from $\{[(c^{1-\sigma} - 1)/(1-\sigma)] - [(\chi l^{1+\varepsilon})/(1+\varepsilon)]\}$.

Figure 1 Time Paths for an Increase in τ_c when $\sigma = 2$

5 The welfare in the steady state is directly calculated from $U^* = \{[(C^*)^{1-\sigma} - 1/(1-\sigma)]$ $[\chi(l^*)^{1+\varepsilon} / (1+\varepsilon)]\}/\rho$. However, the household's discounted sum of utility is based on adding the discounted utility that the household obtains at each point in time, and not just in the steady state.

6 In the Appendix, we have proved that the long-run equilibrium is a saddle, and have also provided the calculation of the transitional dynamics.



Note: Please see the explanations in Figure 1.

Figure 2 Time Paths for an Increase in τ_c When $\sigma = 0.25$

We first discuss the case where the IES for consumption is not large (is less than 1), i.e., $\sigma = 2$. At the point in time at which the consumption tax rate increases, the capital stock is not changed due to the fact that capital is predetermined, while consumption jumps down since the relative price of consumption increases. Leisure is substituted for consumption. Thus labor also jumps down at the beginning, as does the output production. That is, consumption and investment which can increase the capital stock will continue to decrease until they converge in the long-run steady state.

Lower consumption reduces the household's utility, but higher leisure increases it. We obtain that the household's utility does increase in the short run. Since the utility at the beginning is discounted less than that in the future, if the effect of the higher utility in the short run dominates the effect of the lower utility in the long run, the household may feel happier, i.e., the lifetime welfare is increased, under a higher tax rate for consumption. To describe the above effect, we calculate the welfare cost of the consumption tax. The welfare cost is measured in terms of consumption equivalence which is the percentage consumption needed to achieve the initial welfare level, i.e., the initial long-run U under $\tau_c = 5\%$. Thus the positive consumption equivalence means that the policy has a welfare cost, while the negative consumption equivalence implies that the policy has a welfare gain. We obtain that the welfare cost of an increase in the tax rate for consumption (from 5% to 8%) is 0.0620%. This implies that a higher tax rate for consumption does reduce the household's welfare even when we consider the transitional dynamics.

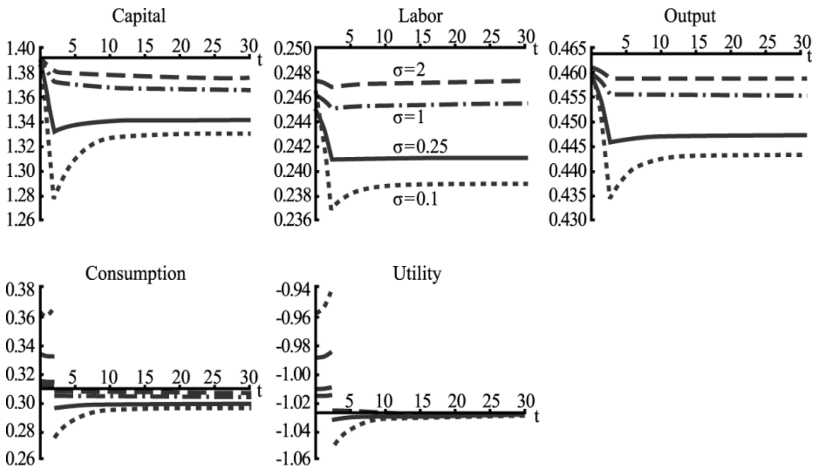
We now discuss the case where the IES for consumption is large (larger than 1), i.e., $\sigma = 0.25.$ The time paths of capital, labor and output are similar to those when $\sigma = 2.$ As for the time path of consumption, there are two effects at work. The first effect is that the relative price of consumption increases, so that the household uses leisure to replace consumption. The other effect results from the higher IES for consumption, and thus the household uses current consumption to replace future consumption. Differing from the case where the IES for consumption is small, the second effect now dominates. Consumption jumps up and is higher than its initial value. Higher consumption and leisure increase the utility at the beginning.

However, when the household uses current consumption to replace future consumption, investment decreases, as does capital. Long-run output will decline more, as will consumption in the long term. We obtain that the welfare cost of an increase in the tax rate on consumption (from 5% to 8%) is 0.1881%, which is larger than that under $\sigma = 2.$

As the IES for consumption may influence the intertemporal substitution effects between consumption and investment (future consumption), and the trade-off between consumption and leisure, the consumption equivalence may become positive under the different IES for consumption. We change σ from 0.1 to 100 and calculate the welfare cost under each case. We obtain that the consumption equivalence is always positive no matter what the value σ is. So a permanent increase in the tax rate on consumption always has a welfare cost.

3.4 Announcement Effects

In reality, the government usually declares a policy before implementing it. Now we analyze the announcement effects of consumption taxes. We take the policy in Japan as an example. On September 12, 2013, the government of Japan announced that the tax rate on consumption would be increased from 5% to 8% in April 2014. To be consistent with the actual situation above, we assume that the initial time is September 12, 2013, and that the consumption tax will be increased after two periods (quarters). The time paths of the related variables according to the announced tax policy are illustrated in Figure 3 (please observe the solid line).[7]



Note: Please see the explanations in Figure 1. In addition, the utility values in all cases are normalized to the initial value under $\sigma = 0.25$.

Figure 3 Announcement Effects of Consumption Taxes

Similar to the result in the above section in which the government executes the policy without announcing it in advance, consumption jumps up and is higher than at its initial steady state in the short run. Intuitively, the households expect that the consumption tax will increase two periods later. Thus they use current consumption to replace future consumption, i.e., they increase consumption expenditure in advance. However, in fact the consumption tax does not rise yet, and the households replace investment with consumption. That is why capital declines at first. In addition, the households substitute leisure for consumption, and thus labor

[7] Since we take the policy in Japan as an example, we use $\sigma = 0.25$ as the initial IES for consumption. However, our results still hold if we use other values of σ.

jumps down at the beginning. Since the capital stock is not changed due to the fact that capital is predetermined, output jumps down at first due to the lower labor.

When the consumption tax increases after two periods, consumption jumps down due to its higher relative price. Since investment (capital) has declined excessively in advance, it thus increases when the consumption tax increases, and will converge to its steady state which is lower than its initial value in the end. The latter result occurs because the higher tax rate for consumption eventually hurts the long-run capital, labor and output.

Similarly, we obtain that the household’s utility does increase in the short run. By comparing Figure 3 with Figure 2, we obtain that the increasing level of utility in the short run in Figure 3 is larger than that in Figure 2. Thus the likelihood that the effect of the higher utility in the short run will dominate the effect of the lower utility in the long run is higher. We calculate the welfare cost of the announced consumption tax and obtain that the consumption equivalence is 0.2166%. There is still a welfare cost under that announced policy, and the cost is greater than that under the policy where no announcement is made in advance.

Intuitively, when the government proclaims that the tax rate on consumption will be increased in advance, the tax rate is not changed immediately, and the intertemporal substitution effect is larger than that without an announcement beforehand. That is why the increase in the level of consumption in the short run in Figure 3 is greater than that in Figure 2. This also implies that the initial investment (capital) needs to be reduced more, which is unfavorable for output production, as is the longer-term consumption and utility. So the welfare cost of the announced consumption tax is greater than that of the non-announced policy.

In order to investigate the intertemporal substitution effects of consumption taxes in depth, we also check the impact of different IES for consumption. The time paths of related variables under different σ are illustrated in Figure 3, and the related welfare costs are listed in Table 1 (please see the second row). We obtain that the welfare cost is reduced as σ increases, or as the IES for consumption declines.

Intuitively, the households are more willing to engage in intertemporal substitution under a higher IES for consumption. In other words, the level of intertemporal substitution is smaller under a lower IES for consumption. Figure 3 shows that the fluctuations in capital, labor, output and consumption are smaller under a lower IES for consumption (a higher σ). In particular, the reductions in those variables are smaller under a higher σ. Thus the utility (welfare) is higher under a higher σ. That is why we obtain that the welfare cost is reduced as σ increases. In our simulation, the announced consumption tax always has a welfare cost no matter what the value of σ is.

Table 1 Welfare Costs of Announced Consumption Taxes

Period	$\sigma = 0.1$	$\sigma = 0.25$	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
1	0.2535	0.2042	0.1543	0.1041	0.0633	0.0355
2	**0.2871**	**0.2166**	**0.1599**	**0.1065**	**0.0642**	**0.0359**
3	0.3127	0.2258	0.1640	0.1082	0.0649	0.0361
4	0.3304	0.2324	0.1669	0.1093	0.0653	0.0363

Note: The welfare costs in terms of consumption equivalence are presented as percentages (%). The period refers to the duration between the time point of the policy's announcement and that of the policy's implementation.

Figure 3 and also Figure 2 imply that if the government declares the policy much earlier, the period in which the short-run utility (consumption) is higher than its initial level is longer, while the longer-term utility is lower due to the lower capital and output. If the former effect dominates, the welfare cost of the announced consumption tax is smaller when the duration between the time point of the policy's announcement and that of the policy's implementation increases, but the welfare cost is higher if the latter effect dominates. We analyze the effect of the above-mentioned duration on the welfare cost, and the results are enumerated in Table 1. The numerical exercises imply that the welfare cost of the announced consumption tax is larger when the government declares the policy much earlier. The above results show that the lower the IES for consumption or the shorter the period between the time of the policy's announcement and that of its implementation, the lower the welfare cost is.

As regards the actual situation in reality, Fuse (2004) has estimated the IES for consumption in Japan and has found that it is around 4, i.e., σ is around 0.25. According to the analysis in this paper, an increase in the tax rate on consumption reduces the output produced and the household's welfare when σ is small. Thus the policy of increasing the consumption tax may have a welfare cost in Japan. In addition, as for the actual behavior of consumption, the percentage changes in consumption from the same quarter of the previous year from the fourth quarter of 2013 to the first quarter of 2015 in Japan are 2.5%, 4.7%, 0.2%, −0.3%, −0.2% and −2.9%, respectively.[8] Our simulation results are consistent with the actual data. During the period between the time of the policy's announcement and that of its implementation, consumption increases, while consumption declines after it is implemented.

Briefly, if the government wants to stimulate the short-run consumption, an announced increasing consumption tax may be a good policy. However, it will harm the household's lifetime welfare, especially in a country with a high IES for consumption.

4. Robustness Checks

To check the robustness of our results, we also analyze the welfare cost of the announced consumption tax when there is no lump-sum transfer, there is no labor-leisure trade-off, under an endogenous growth model, and under different utility functions.

4.1 No Lump-Sum Transfer

The purpose behind increasing the consumption tax in Japan is not only to deal with the government deficit, but also to finance the payment of Social Security. The Japanese government announced that the increase in the consumption tax would be used to finance pensions, health care, elderly

[8] Data resource: http://www.esri.cao.go.jp/index-e.html.

care, fertility decline, and so on. Granting the benefit of Social Security to households is like returning the taxes levied to the households. That is why we use lump-sum transfers which are equal to the levied consumption taxes in the benchmark model.

Now we check the results when there is no lump-sum transfer, i.e., the levied consumption taxes are used by the government. Therefore, the goods market clearance condition in equilibrium is changed to: $\dot{k} = f(k,l) - \delta k - (1+\tau_c)c$. In addition, the steady-state values of consumption and capital become:

$$c^* = \left[\frac{\rho+\delta}{(1-\alpha)} - \delta\right]\frac{k^*}{1+\tau_c}, \tag{12b'}$$

$$k^* = \left\{\left[\frac{\rho+\delta}{(1-\alpha)} - \delta\right]^{-\sigma} \frac{\alpha A}{\chi(1+\tau_c)^{1-\sigma}}\left[\frac{(1-\alpha)A}{\rho+\delta}\right]^{\frac{1-\alpha+\varepsilon}{\alpha}}\right\}^{\frac{1}{\sigma+\varepsilon}}. \tag{12c'}$$

Other equations are the same as those in the benchmark model.

Comparing (12b) and (12b') indicates that the consumption here is lower, as is the utility level. Thus the welfare costs under an increasing consumption tax should be larger than those in the benchmark model. Besides, (12c') shows that changing the tax rate for consumption does not influence the values of capital, labor and output when $\sigma = 1$. That is, only consumption is reduced due to a higher τ_c when $\sigma = 1$. Therefore, the welfare cost under $\sigma = 1$ should be at its lowest level in this case without a lump-sum transfer.

In order to more accurately measure the welfare cost, we use the same steps and setting in the benchmark model to calibrate and simulate the model here. The results are listed in Table 2. The numerical exercises support our theoretical inferences. The welfare costs are somewhat higher than those in our benchmark model, and those under $\sigma = 1$ are lower than others under $\sigma \neq 1$. In addition, since τ_c has a positive (negative) effect on capital when $\sigma > (<) 1$, and always has a negative effect on consumption, the influence of τ_c on the welfare cost is not monotone when σ increases.

Table 2 Welfare Costs in the Economy without Lump-Sum Transfers

Period	$\sigma = 0.1$	$\sigma = 0.25$	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
1	2.8629	2.8482	2.8341	2.8283	2.8464	2.9145
2	**2.8621**	**2.8278**	**2.8081**	**2.7998**	**2.8165**	**2.8830**
3	2.8560	2.8059	2.7818	2.7715	2.7871	2.8519
4	2.8439	2.7828	2.7553	2.7436	2.7582	2.8213

Note: Please see the explanations in Table 1.

4.2 No Labor-Leisure Trade-Off

To clarify the pure effects of intertemporal substitution, we simplify the model to that without a labor-leisure trade-off, i.e., the household only considers consumption in utility. The utility and production functions now become $u(c) = (c^{1-\sigma} - 1)/(1-\sigma)$ and $f(k) = Ak^{1-\alpha}$, respectively. Thus, given τ_c, we can use the following two equations to determine the time paths of c and k: $\dot{k} = f(k) - \delta k - c$ and $\dot{c} = -u_c(c)/u_{cc}(c)[f_k(k) - \delta - \rho]$.

In the steady state, we have $\dot{k} = \dot{c} = 0$, and thus $k^* = [(1-\alpha)A/(\rho+\delta)]^{1/\alpha}$ and $c^* = [(p+\delta)/(1-\alpha) - \delta]k^*$. A change in the consumption tax does not affect the long-run consumption, capital and output, while it influences the time paths along the transitional dynamics. By using the same steps and setting in the benchmark model, we can calibrate and simulate the welfare costs in this model which are listed in Table 3.

Table 3 Welfare Costs in the Model without a Labor-Leisure Trade-Off

Period	$\sigma = 0.1$	$\sigma = 0.25$	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
1	0.0229	0.0081	0.0033	0.0013	0.0005	0.0001
2	**0.0489**	**0.0158**	**0.0064**	**0.0025**	**0.0009**	**0.0003**
3	0.0690	0.0225	0.0092	0.0036	0.0014	0.0005
4	0.0827	0.0279	0.0115	0.0046	0.0018	0.0007

Note: Please see the explanations in Table 1.

From Table 3 it can be inferred that the lack of a labor-leisure trade-off does not affect the main results in this paper. The welfare cost is reduced as the IES for consumption declines, and is increased as the period between the time of the policy's announcement and that of the policy's implementation lengthens.

4.3 Endogenous Growth Model

In the benchmark model, we used a standard Neoclassical growth model. Now we will check the results in the endogenous growth setup. According to King et al. (1988), the production function is now: $f(k, xl) = A(xl)^{\alpha} k^{1-\alpha}$, where xl is referred to as effective labor units, x is the labor productivity with the growth rate at $\dot{x}/x = g > 0.$ Besides, according to King and Rebelo (1999), the IES for consumption must be unity in order to be consistent with a balanced growth path (hereafter, BGP) in the separable utility. That is, we set $\sigma = 1.$

In the Appendix, we have proved that k/x, c/x and λx are constant along the BGP. Define q and z as $q \equiv k/x$ and $z \equiv \lambda x.$ We can simplify the equilibrium conditions by transforming them into a two-dimensional dynamic system with vector (q, z) as follows:

$$\dot{q} = A\left(\frac{\alpha A}{\chi}\right)^{\frac{\alpha}{1-\alpha+\varepsilon}} Z^{\frac{\alpha}{1-\alpha+\varepsilon}} q^{\frac{(1-\alpha)(1+\varepsilon)}{1-\alpha+\varepsilon}} - (\delta + g)q - \frac{1}{z(1+\tau_c)}, \tag{13a}$$

$$\dot{z} = z\left[\rho + \delta + g - (1-\alpha)A\left(\frac{\alpha A}{\chi}\right)^{\frac{\alpha}{1-\alpha+\varepsilon}} Z^{\frac{\alpha}{1-\alpha+\varepsilon}} q^{\frac{-\alpha\varepsilon}{1-\alpha+\varepsilon}}\right], \tag{13b}$$

The time paths of c/x and l are $c/x = 1/[z(1+\tau_c)]$ and $l = [(\alpha A/\chi)\, zp^{1-\alpha}]^{1/(1-\alpha+\varepsilon)}$, respectively.

Along the BGP, we have $\dot{q} = \dot{z} = 0,$ along with the above relationships, and obtain the following long-run values:

$$l^* = \left[\frac{\rho + \delta + g}{(1-\alpha)A} \right]^{\frac{1}{\alpha}} q^*, \tag{14a}$$

$$\left(\frac{c}{x}\right)^* = \left[\frac{\rho + \delta + g}{(1-\alpha)} - \delta - g \right] q^*, \tag{14b}$$

$$q^* = \left\{ \left[\frac{\rho + \delta + g}{(1-\alpha)} - \delta - g \right]^{-1} \frac{\alpha A}{\chi(1+\tau_c)} \left[\frac{(1-\alpha)A}{\rho + \delta + g} \right]^{\frac{1-\alpha+\varepsilon}{\alpha}} \right\}^{\frac{1}{1+\varepsilon}}. \tag{14c}$$

Consumption, capital and output grow at the rate g in the long term.

To specifically calculate the welfare cost, we use the same steps and setting, and assume that the annual growth rate of the economy is 2%, i.e., g = 0.005, and normalize the initial labor productivity at $x(0)$ = 1, to calibrate and simulate the model. The welfare costs are exactly the same as those in the fifth column in Table 1 (the case $\sigma = 1$). One of the main results in this paper still holds in the endogenous growth model: the welfare cost declines when the period between the time of the policy's announcement and that of its implementation is shortened.

Since the above utility function can not estimate the effects of intertemporal substitution, to investigate the above effects in the endogenous growth model, we now simplify our utility function to that without a labor-leisure trade-off like that in Section 4.2, i.e., $u(c) = (c^{1-\sigma} - 1)/(1-\sigma)$. Thus, the production function now becomes: $f(k,x) = A(x)^{\alpha} k^{1-\alpha}$. Similarly, the two-dimensional dynamic system becomes

$$\dot{q} = Aq^{1-\alpha} - (\delta + g)q - \left[z(1+\tau_c) \right]^{\frac{-1}{\sigma}}, \tag{13a'}$$

$$\dot{z} = z\left[\rho + \delta + \sigma g - (1-\alpha)Aq^{-\alpha} \right]. \tag{13b'}$$

The time path of c/x now is $c/x = [z(1+\tau_c)]^{\frac{-1}{\sigma}}$.

Along the BGP, we obtain the following long-run values:

$$\left(\frac{c}{x}\right)^* = \left[\frac{\rho+\delta+\sigma g}{(1-\alpha)} - \delta - g\right] q^*. \tag{14b'}$$

$$q^* = \left[\frac{(1-\alpha)A}{\rho+\delta+\sigma g}\right]^{\frac{1}{\alpha}}. \tag{14c'}$$

Similar to that in Section 4.2, a change in the consumption tax does not affect the long-run c/x, k/x and y/x, but it influences the time paths along the transitional dynamics. Using the same calibration method, we can obtain the simulation results which are listed in Table 4. Table 4 shows that our main results still hold under an endogenous growth model.

Table 4 Welfare Costs in the Endogenous Growth Model

Period	$\sigma=0.1$	$\sigma=0.25$	$\sigma=0.5$	$\sigma=1$	$\sigma=2$	$\sigma=4$
1	0.0045	0.0038	0.0030	0.0022	0.0018	0.0016
2	**0.0081**	**0.0076**	**0.0056**	**0.0041**	**0.0033**	**0.0028**
3	0.0161	0.0112	0.0079	0.0057	0.0045	0.0038
4	0.0237	0.0144	0.0099	0.0071	0.0054	0.0045

Note: Please see the explanations in Table 1.

4.4 Different Elasticities of Substitutability between Consumption and Leisure

In the above estimations, we only use two kinds of utility functions: one is separable between consumption and leisure (labor), and the other only consists of consumption. To investigate the interactive effect between consumption and leisure, we now focus on the different elasticities of substitutability between these two variables.

We follow Keller (1976) and Lu et al. (2011), and use a constant elasticity of substitutability (CES) utility function: $u(c,1-l)=[bc^{\eta}+(1-b)(1-l)^{\eta}]^{1/\eta}$, where $\eta=(\varepsilon-1)/\varepsilon$ and $\varepsilon>0$ is the elasticity of substitutability (hereafter, ES) and b is the intensity of consumption in utility relative to leisure.

Thus, given τ_c, we can use the following two equations to determine the time paths of k and $\lambda: \dot{k} = f[k, l(k,\lambda)] - \delta k - c(k,\lambda)$ and $\dot{\lambda} = \lambda[\delta + p - (1-\alpha)Al(k,\lambda)^{\alpha} k^{-\alpha}]$. In addition, the time paths of l and c are as follows: $l(k,\lambda) = [(1/\lambda)^{\eta/(\eta-1)} - b^{1/(1-\eta)}(1+\tau_c)^{\eta/(\eta-1)}]^{(1-\eta)/\eta(1-\alpha)}(\alpha A)^{1/(1-\alpha)}(1-b)^{-1/\eta(1-\alpha)}k$ and $c(k,\lambda) = [1 - l(k,\lambda)]\{[(1+\tau_c)/\alpha Al(k,l)^{\alpha-1}k^{1-\alpha}](1-b)/b)\}^{1/(\eta-1)}$.

In the steady state, the long-run values of c and l are the same as those in (12b) and (12a), respectively, and that of k is as follows:

$$k^* = \frac{\left\{\frac{1-b}{b}\frac{1+\tau_c}{\alpha A}\left[\frac{\rho+\delta}{(1-\alpha)A}\right]^{\frac{1-\alpha}{\alpha}}\right\}^{\frac{1}{\eta-1}}}{\frac{\rho+\delta}{(1-\alpha)} - \delta + \left\{\frac{1-b}{b}\frac{1+\tau_c}{\alpha A}\left[\frac{\rho+\delta}{(1-\alpha)A}\right]^{\frac{1-\alpha}{\alpha}}\right\}^{\frac{1}{\eta-1}}\left[\frac{\rho+\delta}{(1-\alpha)A}\right]^{\frac{1}{\alpha}}}. \quad (12c'')$$

Similar to the result in the benchmark model, an increase in the consumption tax decreases the steady-state capital stock, and thus consumption, labor, and output also decline in the long run.

Using the same calibration method and assuming the value of the ES in consumption to be $\varepsilon = 2.5$ which is that used in Lu et al. (2011), i.e., $\eta = 0.6$, we can calibrate the intensity of consumption in utility relative to leisure at b=0.3834, and the initial values of all other variables are the same as those in the benchmark model (c=0.3107, k = 1.3914, and y= 0.4638, besides U=56.3753). When τ_c is increased from 5% to 8%, we obtain that $c^* = 0.2946$, $l^* = 0.2370$, k^* = 1.3192, $y^* = 0.4397$, and U^*= 56.2937, and all variables are reduced as τ_c increases.

As for the welfare cost along the transitional dynamics, we obtain that under the policy in Japan as being 0.3232%. We also estimate the welfare costs under the different ES between consumption and leisure and under the different periods between the time of the announcement and that of the policy's implementation. The simulation results are reported in Table 5.

Table 5 supports one of our main results that the welfare cost increases as the above-mentioned period lengthens. Besides, Table 5 also infers that the welfare cost increases together with the ES between consumption and leisure. Intuitively, the higher the ES, the more the households will be willing to substitute leisure for consumption when the relative price of consumption increases. That is, when τ_c increases, the households use leisure to replace consumption. Therefore, consumption and labor decline, as do capital and output. So long-run consumption also declines, which will reduce the welfare in the long run. However, higher leisure increases the household's utility. Our numerical exercises show that the former effect dominates. Thus, the welfare cost rises as the ES between consumption and leisure increases.

Table 5 Welfare Costs under Different ES between Consumption and Leisure

Period	$\varepsilon = 0.5$	$\varepsilon = 1.5$	$\varepsilon = 2.5$	$\varepsilon = 3.5$	$\varepsilon = 4.5$
1	0.0646	0.1841	0.3023	0.4187	0.5328
2	**0.0705**	**0.1966**	**0.3232**	**0.4500**	**0.5769**
3	0.0745	0.2044	0.3358	0.4686	0.6027
4	0.0771	0.2092	0.3434	0.4796	0.6179

Note: Please see the explanations in Table 1.

5. Concluding Remarks

This paper investigates the effect of an increase in the tax rate on consumption both in the steady state and along the transitional dynamics. In the latter case, we consider two situations. One is where the government executes the policy without making an announcement in advance. The other is where the government announces the policy before implementing it, which gives rise to the so-called announcement effect.

Our results are as follows. First, an increase in the tax rate on consumption eventually hurts the long-run capital, labor, output and consumption. A higher tax rate for consumption lowers the household’s welfare if we only investigate the steady state. Next, even when we consider the transitional dynamics, if the government implements the policy without declaring it in advance, a higher tax rate on consumption will always entail a welfare cost. Finally, if we are concerned with an announcement effect of consumption taxes, the welfare cost is reduced as the IES for consumption declines, and it is increased as the period between the time of the policy’s announcement and that of the policy’s implementation is lengthened.

Our paper can simulate the actual situation in Japan. On September 12, 2013, the government of Japan announced that the tax rate on consumption would be increased two quarters later. An announced consumption tax can stimulate the short-run consumption. Our simulation results are consistent with the actual data in Japan. In addition to the policy actually impacting Japan’s economy in reality, our paper further indicates that people in Japan may feel less happy due to the lower welfare under this policy since the IES for consumption in Japan is high.

Moreover, we also perform a lot of robustness checks. Our results still hold when there is no labor-leisure trade-off, under the endogenous growth model, and also under different utility functions.

The contribution of this paper is that we can provide advice to the government. When the government plans to implement certain policies, especially to enhance the tax rates, if it wants to stimulate the short-run consumption (or other macroeconomic variables), an announced policy may be a good method. However, it may harm the household’s lifetime welfare, especially in a country with a high IES for consumption.

Appendix

This appendix presents technical details regarding the transitional dynamics in all cases. To be consistent with all conditions in equilibrium at any given point in time, we follow the method used in Djajić (1987), Turnovsky and Fisher (1995) and their subsequent studies. The shadow price (λ) and capital are continuous, even at the point where the government implements the policy. Using this characteristic, we can determine the equilibrium paths of k and λ. The time paths of c and l can be determined by their related equilibrium conditions, as can those of output and utility.

Appendix 1 Technical Details under the benchmark Model

Using our functional form and (3a) and (3b), along with (7b), the time paths of consumption and labor are functions of k and λ as follows:

$$c = \lambda^{\frac{-1}{\sigma}} (1+\tau_c)^{\frac{-1}{\sigma}}, \tag{A1a}$$

$$l = \left(\frac{\alpha A}{\chi}\right)^{\frac{1}{1-\alpha+\varepsilon}} \lambda^{\frac{1}{1-\alpha+\varepsilon}} k^{\frac{1-\alpha}{1-\alpha+\varepsilon}}. \tag{A1b}$$

Once we determine the time paths of k and λ, those of c and l can also be determined by (A1a) and (A1b).

Next, we can use (3c) and (9), along with (A1a), (A1b) and (7a), to determine the time paths of k and λ. We take Taylor's expansion of system (3c) and (9) in the neighborhood of the steady state and obtain

$$\begin{bmatrix} \dot{k} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} k - k^* \\ \lambda - \lambda^* \end{bmatrix}, \tag{A2}$$

where

$$J_{11} = \frac{(1-\alpha)(\varepsilon+1)}{1-\alpha+\varepsilon} A\left(\frac{\alpha A}{\chi}\right)^{\frac{\alpha}{1-\alpha+\varepsilon}} (\lambda^*)^{\frac{\alpha}{1-\alpha+\varepsilon}} (k^*)^{\frac{(1-\alpha)(\varepsilon+1)}{1-\alpha+\varepsilon}-1} - \delta > 0,$$

$$J_{12} = \frac{\alpha A}{1-\alpha+\varepsilon}\left(\frac{\alpha A}{\chi}\right)^{\frac{\alpha}{1-\alpha+\varepsilon}} (\lambda^*)^{\frac{\alpha}{1-\alpha+\varepsilon}-1} (k^*)^{\frac{(1-\alpha)(\varepsilon+1)}{1-\alpha+\varepsilon}} + \frac{(1+\tau_c)^{-\frac{1}{\sigma}} (\lambda^*)^{-\frac{1}{\sigma}-1}}{\sigma} > 0,$$

$$J_{21} = \frac{\alpha\varepsilon}{1-\alpha+\varepsilon}(1-\alpha)A\left(\frac{\alpha A}{\chi}\right)^{\frac{\alpha}{1-\alpha+\varepsilon}} (\lambda^*)^{\frac{\alpha}{1-\alpha+\varepsilon}+1} (k^*)^{\frac{-\alpha\varepsilon}{1-\alpha+\varepsilon}-1} > 0,$$

$$J_{22} = -\frac{\alpha}{1-\alpha+\varepsilon}(1-\alpha)A\left(\frac{\alpha A}{\chi}\right)^{\frac{\alpha}{1-\alpha+\varepsilon}} (\lambda^*)^{\frac{\alpha}{1-\alpha+\varepsilon}} (k^*)^{\frac{-\alpha\varepsilon}{1-\alpha+\varepsilon}} < 0.$$

The equilibrium dynamic system involves one state variable whose initial value is predetermined and one costate variable which may adjust instantaneously. The dynamic equilibrium path toward a steady state is unique if the characteristic function in association with the Jacobian matrix in (A2) has only one negative eigenvalue. It is easy to calculate that the determinant of the Jacobean matrix in (A2) is negative, and thus it has one negative eigenvalue and one positive eigenvalue. Thus the long-run equilibrium is a saddle.

Appendix 1.1 Transition Dynamics in which the Government Implements the Policy without Declaring It in Advance

Let $\mu_i, i=1,2,$ be the eigenvalue of the Jacobean matrix in (A2) and μ_1 be the negative eigenvalue. Then, the equilibrium time paths for the capital stock and the shadow price are as follows:

$$k(t) = k^* + B_1 v_{11} e^{\mu_1 t} + B_2 v_{12} e^{\mu_2 t},$$

$$\lambda(t) = \lambda^* + B_1 v_{21} e^{\mu_1 t} + B_2 v_{22} e^{\mu_2 t},$$

where $v_{ji}, j=1,2,$ is the eigenvector corresponding to $\mu_i, i=1,2,$ and coefficient $B_i, i=1,2,$ is determined by the boundary conditions.

To determine B_i, note that both variables must converge to their steady-state values when $t \to \infty$. As μ_2 is positive, this is possible only if $B_2 = 0$. As a result, the equilibrium time paths become

$$k(t) = k^* + B_1 v_{11} e^{\mu_1 t}, \tag{A3a}$$

$$\lambda(t) = \lambda^* + B_1 v_{21} e^{\mu_1 t}. \tag{A3b}$$

Let $k'(k^*)$ be the steady state before (after) an increase in the tax rate for consumption. As capital is not affected at $t = 0$, (A3a) must satisfy the following condition

$$k' = k^* + B_1 v_{11}. \tag{A3c}$$

The above relationship determines coefficient B_1. If we substitute B_1 into (A3b) for time 0, we obtain $\lambda(0)$ which is adjusted instantaneously to the saddle arm. Note that $\lambda(0)$ is the level of the shadow price at the moment that the tax rate for consumption increases. Using B_1 and (A3a)-(A3b), we obtain the equilibrium time paths of *k* and λ at any point in time. Then the time paths of *c*, *l*, *y* and $u(c,l)$ can be derived from (A1a), (A1b), (5') and (1'), respectively.

Appendix 1.2 Transition Dynamics in which the Government Announces the Policy before Implementing It

The government announces increasing the tax rate on consumption at time 2. The time paths of key variables differ before and after the policy realized at time 2. Using the same method as in Appendix 1.1, the equilibrium time paths of key variables before time 2 are

$$\begin{aligned} k(t) &= k' + D_1 v'_{11} e^{\mu'_1 t} + D_2 v'_{12} e^{\mu'_2 t} \\ \lambda(t) &= \lambda' + D_1 v'_{21} e^{\mu'_1 t} + D_2 v'_{22} e^{\mu'_2 t} \end{aligned} \quad \text{when } t = 0^+ \sim 2^-, \tag{A4a}$$

and the equilibrium time paths after time 2 are

$$\begin{aligned} k(t) &= k^* + D_3 v_{11} e^{\mu_1 t} \\ \lambda(t) &= \lambda^* + D_3 v_{21} e^{\mu_1 t} \end{aligned} \quad \text{when } t \geq 2^+, \tag{A4b}$$

where time 0^+ indicates the moment when the policy is announced, and time 2^- and 2^+ are the moments before and after the policy is actually realized.

In the notation, a variable with a superscript $'$ denotes a steady-state value under $\tau_c = 5\%$, while that with * stands for a steady-state value under $\tau_c = 8\%$; μ_i is the eigenvalue of the Jacobean matrix in (A2) associated with $\tau_c = 8\%$ while μ_i' is the eigenvalue of the Jacobean matrix (A2) associated with $\tau_c = 5\%$. The eigenvector for μ_i is v_{ji} and the eigenvector for μ_i' is v_{ji}'. The three coefficients, D_1, D_2 and D_3, are determined as follows.

First, at $t = 0$ when the policy is announced, capital does not change. We thus obtain

$$k' = k' + D_1 v_{11}' + D_2 v_{12}'. \tag{A5a}$$

Moreover, right before and after the time when the policy is realized at time 2, the continuity of the equilibrium time paths implies that

$$k' + D_1 v_{11}' e^{2\mu_1'} + D_2 v_{12}' e^{2\mu_2'} = k^* + D_3 v_{11} e^{2\mu_1}, \tag{A5b}$$

$$\lambda' + D_1 v_{21}' e^{\mu_1' 2} + D_2 v_{22}' e^{\mu_2' 2} = \lambda^* + D_3 v_{11} e^{\mu_1 2}. \tag{A5c}$$

Equations (A5a)-(A5c) determine the values for coefficients D_1, D_2 and D_3. With these values of the coefficients, we then use (A4a) and (A4b) to obtain the equilibrium time paths of k and λ at any point in time. Then the time paths of c, l, y and $u(c,l)$ can be derived from (A1a), (A1b), (5') and (1'), respectively.

Appendix 2 Technical Details under the Model without a Lump-Sum Transfer

In this case, the time paths of consumption and labor are the same as (A1a) and (A1b). However, the goods market clearance condition becomes:

$$\dot{k} = f(k,l) - \delta k - (1+\tau_c)^{1-\frac{1}{\sigma}} \lambda^{\frac{-1}{\sigma}}. \tag{A6}$$

In addition, except for J_{12} which we list below, the values of all other elements of the Jacobian matrix are the same as those in (A2).

$$J_{12} = \frac{\alpha A}{1-\alpha+\varepsilon}\left(\frac{\alpha A}{\chi}\right)^{\frac{\alpha}{1-\alpha+\varepsilon}} (\lambda^*)^{\frac{\alpha}{1-\alpha+\varepsilon}-1} (k^*)^{\frac{(1-\alpha)(\varepsilon+1)}{1-\alpha+\varepsilon}} + \frac{(1+\tau_c)^{1-\frac{1}{\sigma}} (\lambda^*)^{-\frac{1}{\sigma}-1}}{\sigma} > 0$$

Using the same method in Section 6.1, we can determine the time paths of k and λ at any point in time. Then the time paths of c, l, y and $u(c,l)$ can be derived from (A1a), (A1b), (5') and (1'), respectively.

Appendix 3 Technical Details under the Model without a Labor-Leisure Trade-Off

When there is no labor-leisure trade-off, the household's necessary conditions become:

$$c^{-\sigma} = \lambda(1+\tau_c), \tag{A7a}$$

$$-\lambda(r-\delta) = \dot{\lambda} - \rho\lambda. \tag{A7b}$$

The firm's necessary conditions are changed to $\omega = \alpha A k^{1-\alpha}$ and $r = (1-\alpha)Ak^{-\alpha}$.

Thus the goods market clearance condition in equilibrium is changed to $\dot{k} = Ak^{1-\alpha} - \delta k - c$, where $c = \lambda^{-1/\alpha}(1+\tau_c)^{-1/\alpha}$ according to (A7a). Besides, $\dot{\lambda} = \lambda[\rho + \delta - (1-\alpha)Ak^{-\alpha}]$. Using the above two equations, we can calculate the Jacobian matrix, (A2), in this case, where $J_{11} = (1-\alpha)A(k^*)^{-\alpha} - \delta > 0$, $J_{12} = (1/\sigma)(1-\tau_c)^{-1/\sigma}(\lambda^*)^{(-1/\sigma)-1} > 0$, $J_{21} = \lambda^* \alpha(1-\alpha)A(k^*)^{-\alpha-1} > 0$, and $J_{22} = 0$.

The determinant of the Jacobean matrix in (A2) is negative, and thus the long-run equilibrium is also a saddle. Using the same method in

Appendix 1, we can determine the time paths of k and λ at any point in time. Other time paths can be determined by the following equations:

$$c = \lambda^{\frac{-1}{\sigma}} (1+\tau_c)^{\frac{-1}{\sigma}},$$

$$y = Ak^{1-\alpha},$$

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}.$$

Appendix 4 Technical Details under an Endogenous Growth Model

Appendix 4.1 Benchmark Model with an Endogenous Growth Production Function

In this case, the household's necessary conditions become:

$$c^{-1} = \lambda(1+\tau_c), \tag{A8a}$$

$$\chi l^{-\varepsilon} = \lambda w x, \tag{A8b}$$

$$-\lambda(r-\delta) = \dot{\lambda} - \rho\lambda. \tag{A8c}$$

The firm's necessary conditions are changed to $w = \alpha A(xl)^{\alpha-1} k^{1-\alpha}$ and $r = (1-\alpha)A(xl)^{\alpha} k^{-\alpha}$.

Putting r into (A8c) yields $\dot{\lambda}/\lambda = \rho + \delta - (1-\alpha)A(l)^{\alpha} (k/x)^{-\alpha}$. The above equation implies that if the BGP exists, i.e., the growth rate of λ is a constant, k/x is constant along the BGP. Besides, the goods market clearance condition now becomes $\dot{k}/k = Al^{\alpha} (k/x)^{-\alpha} - \delta - [(c/x)/(k/x)]$. Thus c/x is also constant along the BGP. Moreover, (A8a) shows that $1/c/x = \lambda x(1+\tau_c)$, which infers that λx is a constant along the BGP, and this inference is consistent with the implication in (A8b). That is, we define that $q \equiv k/x$ and $z \equiv \lambda x$. (13a) and (13b) can be derived from $\dot{q}/q = \dot{k}/k - \dot{x}/x$ and $\dot{z}/z = \dot{\lambda}/\lambda + \dot{x}/x$, respectively.

We take Taylor's expansion of system (13a) and (13b) in the neighborhood of the BGP and obtain

$$\begin{bmatrix} \dot{q} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} q - q^* \\ z - z^* \end{bmatrix}, \tag{A9}$$

where

$$J_{11} = \frac{(1-\alpha)(\varepsilon+1)}{1-\alpha+\varepsilon} A \left(\frac{\alpha A}{\chi}\right)^{\frac{\alpha}{1-\alpha+\varepsilon}} (z^*)^{\frac{\alpha}{1-\alpha+\varepsilon}} (q^*)^{\frac{(1-\alpha)(\varepsilon+1)}{1-\alpha+\varepsilon}-1} - \delta - g > 0,$$

$$J_{12} = \frac{\alpha A}{1-\alpha+\varepsilon} \left(\frac{\alpha A}{\chi}\right)^{\frac{\alpha}{1-\alpha+\varepsilon}} (z^*)^{\frac{\alpha}{1-\alpha+\varepsilon}-1} (q^*)^{\frac{(1-\alpha)(\varepsilon+1)}{1-\alpha+\varepsilon}} + \frac{1}{(z^*)^2(1+\tau_c)} > 0,$$

$$J_{21} = \frac{\alpha\varepsilon}{1-\alpha+\varepsilon} (1-\alpha) A \left(\frac{\alpha A}{\chi}\right)^{\frac{\alpha}{1-\alpha+\varepsilon}} (z^*)^{\frac{\alpha}{1-\alpha+\varepsilon}+1} (q^*)^{\frac{-\alpha\varepsilon}{1-\alpha+\varepsilon}-1} > 0,$$

$$J_{22} = -\frac{\alpha}{1-\alpha+\varepsilon} (1-\alpha) A \left(\frac{\alpha A}{\chi}\right)^{\frac{\alpha}{1-\alpha+\varepsilon}} (z^*)^{\frac{\alpha}{1-\alpha+\varepsilon}} (q^*)^{\frac{-\alpha\varepsilon}{1-\alpha+\varepsilon}} < 0.$$

The determinant of the Jacobean matrix in (A9) is negative, and thus it has one negative eigenvalue and one positive eigenvalue. Thus the long-run equilibrium is a saddle. Note that only q is a predetermined variable since both k and x are predetermined. Using the same method in Appendix 1.1 and 1.2, we can determine the time paths of q and z at any point in time. The time paths of c/x and l can be determined from (A8a) and (A8b), respectively. Furthermore, $y/x = Al^{\alpha} q^{1-\alpha}$ and $u(c,l) = \ln(c/x) + \ln(x(0)) + gt - \chi l^{1+\varepsilon} / (1+\varepsilon)$.

Appendix 4.2 An Endogenous Growth Model without a Labor-Leisure Trade-Off

We now simplify our model into that without a labor-leisure trade-off, and the household's necessary conditions are the same as (A8a) and (A8c). The firm's necessary conditions are changed to $w = \alpha A (k/x)^{1-\alpha}$ and

$r=(1-\alpha)A(k/x)^{-\alpha}$. Moreover, the goods market clearance condition now becomes $\dot{k}/k=A(k/x)^{-\alpha}-\delta-(c/x)/(k/x)$. Similarly, k/x, c/x and λx are constant along the BGP. (13a') and (13b') can be derived from $\dot{q}/q=\dot{k}/k-\dot{x}/x$ and $\dot{z}/z=\dot{\lambda}/\lambda+\dot{x}/x$, respectively.

Now, the values of the elements in (A9) become $J_{11}=(1-\alpha)A(q^*)^{-\alpha}-\delta-g>0$, $J_{12}=(1/\sigma)(z^*)^{(-1/\sigma)-1}(1-\tau_c)^{-1/\sigma}>0$, $J_{21}=z^*\alpha(1-\alpha)A(q^*)^{-\alpha-1}>0$, and $J_{22}=0$. The determinant of the above Jacobean matrix is negative, and thus the long-run equilibrium is a saddle. Using the same method as in Appendix 1.1 and 1.2, we can determine the time paths of q and z at any point in time. The time paths of c/x can be determined from (A8a). Furthermore, $y/x=Aq^{1-\alpha}$ and $u(c)=\ln(c/x)+\ln(x(0))+gt$.

Appendix 5 Technical Details under Different ES between Consumption and Leisure

In this case, the household's consumption-leisure trade-off condition in equilibrium becomes

$$\frac{bc^{\eta-1}}{(1-b)(1-l)^{\eta-1}}=\frac{1+\tau_c}{\alpha A l^{\alpha-1}k^{1-\alpha}}. \tag{A10a}$$

By combining (3a) and (A10a), we can derive the time paths of labor and consumption which are functions of k and λ as follows:

$$l=\left[\left(\frac{1}{\lambda}\right)^{\frac{\eta}{\eta-1}}-b^{\frac{1}{1-\eta}}(1+\tau_c)^{\frac{\eta}{\eta-1}}\right]^{\frac{1-\eta}{\eta(1-\alpha)}}(\alpha A)^{\frac{1}{1-\alpha}}(1-b)^{\frac{-1}{\eta(1-\alpha)}}k. \tag{A10b}$$

$$c=\left[\frac{1+\tau_c}{\alpha A l^{\alpha-1}k^{1-\alpha}}\frac{1-b}{b}\right]^{\frac{1}{\eta-1}}(1-l). \tag{A10c}$$

Now the values of the elements in (A2) become:

$$J_{11}=\alpha A\left(l^*\right)^{\alpha-1}(k^*)^{1-\alpha}\ \frac{dl}{dk}+(1-\alpha)A\left(l^*\right)^{\alpha}(k^*)^{-\alpha}\ -\delta-\frac{dc}{dk},$$

$$J_{12}=\alpha A\left(l^*\right)^{\alpha-1}(k^*)^{1-\alpha}\ \frac{dl}{d\lambda}-\frac{dc}{d\lambda},$$

$$J_{21}=\left[-\alpha(1-\alpha)A(l^*)^{\alpha-1}(k^*)^{-\alpha}\ \frac{dl}{dk}+\alpha(1-\alpha)A(l^*)^{\alpha}(k^*)^{-\alpha-1}\right]\lambda^*,$$

$$J_{22}=-\alpha\left(1-\alpha\right)A\left(l^*\right)^{\alpha-1}(k^*)^{-\alpha}\ \frac{dl}{d\lambda}\lambda^*,$$

where

$$\frac{dl}{dk}=\frac{\dfrac{(1-\alpha)}{k^*}}{\dfrac{1-\alpha}{l^*}+\dfrac{1-\eta}{1-l^*}+\dfrac{(1-\eta)B_1}{c^*}},$$

$$\frac{dl}{d\lambda}=\frac{\dfrac{(\eta-1)B_2}{c^*}}{\dfrac{1-\alpha}{l^*}+\dfrac{1-\eta}{1-l^*}+\dfrac{(1-\eta)B_1}{c^*}},$$

$$\frac{dc}{dk}=B_1\frac{dl}{dk},\ \frac{dc}{d\lambda}=B_1\frac{dl}{dk}+B_2,$$

$$B_1=\frac{\dfrac{\left(\dfrac{1-\sigma}{\eta}-1\right)(1-b)\eta(1-l^*)^{\eta-1}}{b(c^*)^{\eta}+(1-b)(1-l^*)^{\eta}}}{\left[\dfrac{\left(\dfrac{1-\sigma}{\eta}-1\right)b\eta(c^*)^{\eta-1}}{b(c^*)^{\eta}+(1-b)(1-l^*)^{\eta}}+\dfrac{\eta-1}{c^*}\right]},$$

and

$$B_2 = \frac{\frac{1}{\lambda}}{\left[\frac{\left(\frac{1-\sigma}{\eta}-1\right)b\eta(c^*)^{\eta-1}}{b(c^*)^{\eta}+(1-b)(1-l^*)^{\eta}}+\frac{\eta-1}{c^*}\right]}.$$

In our numerical exercises, the determinant of the Jacobean matrix in (A2) is negative, and thus the long-run equilibrium is also a saddle. Using the same method in Appendix 1, we can determine the time paths of *k* and λ at any point in time. Then the time paths of *l*, *c*, *y* and utility can be derived from (A10b), (A10c), (5') and $u(c,1-l)=[bc^{\eta}+(1-b)(1-l)^{\eta}]^{1/\eta}$, respectively.

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消費稅的宣示效果

盧佳慧

摘　要

本篇文章研究消費稅增加對總體經濟在長期恆定狀態以及傳遞動態的影響。當我們只研究長期恆定狀態時，消費稅增加只會對民眾福利有負面影響。即使我們研究傳遞動態，如果政府執行政策沒有事先宣告，消費稅也只會帶來福利損失。除此之外，若政府在執行政策前事先宣告，本篇文章可以得到當該經濟體的民眾的消費跨期替代彈性較低，福利損失會較少；但若是宣告到執行政策的間隔時間較長，福利損失會增加。本文的結果在無休閒勞動選擇，或是內生成長模型，或是不同效用函數之下皆可成立。

關鍵詞：宣示效果、消費稅、福利損失
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