Information Acquisition in Rent-seeking Contests with a Common Value

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Abstract

This research considers the situation where there are two teams, each of which is composed of a principal and an agent, competing in a contest with a prize that has a common value. The value of the prize is uncertain to the principal and can be either high or low, while the agent knows such information. A principal has two methods to acquire information: self-investigation, where the principal acquires information by herself; and delegation, where she delegates the contest right to the agent who competes on her behalf. We find that when the gap in the prize values is relatively large and the cost of information acquisition is relatively small, both principals adopt self-investigation in equilibrium, while if both the gap in the prize values and the cost of information acquisition are sufficiently large then delegation by one of the principals can be an equilibrium. However, there is no equilibrium when both principals delegate, even though it is socially optimal for them to do so.

Keywords: Rent-Seeking Contests, Information Acquisition, Delegation
JEL Classification: C72, D81, D82

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Received May 7, 2021; revised July 7, 2021; accepted May 27, 2022.

經濟研究 (Taipei Economic Inquiry), 59:2 (2023), 301-345。
臺北大學經濟學系出版

1. Introduction

Rent-seeking contests describe the situation where people make effort in competing with each other for a prize. Many real-world situations can be regarded as rent-seeking contests, such as competition between lobby groups, lawsuits in courts, oil-tract auctions, etc. A workhorse model for analyzing contests is Tullock (1980), which has been followed by a large literature. In Tullock's model, players put forth effort to win the prize, and the winning probability depends on the relative strength of the effort made by players. In some situations, the value of the prize is publicly known, such as in professional sports; while in other cases, the value of the prize is uncertain. For example, lobby groups make contributions to a party hoping to get some benefits in return when it wins the election. However, they do not know how likely that the party will win the election, or how much the future benefit will be. Such information is valuable because players want to make the right decision, so that they can put forth more effort in pursuing a high-value prize and less effort in a low-value one. This paper contributes to this literature by incorporating information acquisition activity in Tullock contests where players have incomplete information about the value of the prize.

In the real world, a principal often delegates the decision-making right to an agent in contests. For example, firms hire purchasing agents to acquire profitable procurement contracts, actors or professional athletes hire sports agents to secure good contracts, and litigants hire lawyers to win lawsuits. However, sometimes, players would instead enter the contest and acquire information by themselves. For example, there is a growing trend of pro se (meaning "for oneself" or "on behalf of oneself") *representation* by athletes in contract negotiations because agents' commissions are rising steeply. Similarly, since legal fees and expenses have continued increasing in recent years, more litigants choose *pro se* representation without hiring an attorney, which means that they have to gather evidence and represent themselves in court. Thus, in this paper, we consider two methods from which the principal can improve the information: "self-investigation," where the principal acquires the information about the value of the prize with a cost and enters the contest on her own, and "delegation," in which the principal delegates the right to an agent who competes in the contest on her behalf.

Consider the situation where there are two teams, each composed of a principal (she) and an agent (he), competing in a contest. The agent knows the value of the prize and so it is beneficial for the principal to delegate. However, in order to induce the agent to make effort, the principal needs to pay the agent a share of the prize. On the other hand, under self-investigation, the principal acquires the information about the prize value at a cost and then enters the contest by herself. The trade-off in determining which method to be adopted in the equilibrium lies between the benefit and the cost of information acquisition. The benefit is derived from making the right effort decision in pursuing the true prize value. If the gap between the high value and the low value of the prize is large, the benefit from acquiring information is also large, and so players have more of an incentive to pursue the high-value prize by making more effort. Thus, acquiring information via self-investigation is worthwhile if the cost of information acquisition is relatively small. On the other hand, if the cost of information is relatively large, information acquisition via self-investigation is not worth doing, and so delegation or acquiring no information can be more preferable.

We find that when the gap in the prize values is relatively small and the cost of information acquisition is relatively large, neither principal engages in acquiring information in the equilibrium. One of the principals begins to acquire information via self-investigation when the cost of information and the gap in the prize values are intermediate. When the gap in the prize values is relatively large and the cost of information is relatively small, both principals adopt self-investigation in the equilibrium. On the other hand, delegation by one principal can be an equilibrium when both the gap in the prize values and the cost of information are sufficiently large. One particularly interesting finding is that there is no equilibrium where both principals delegate, although it is socially optimal for them to do so. This implies that, from the society's point of view, the principals delegate too little in the equilibrium, in the sense that they keep too much authority by entering the contest by themselves while they should have let the agents compete for them.

The rest of the paper is organized as follows. Section 2 reviews the related literature. In Section 3, we present the baseline model. Section 4 characterizes the optimal effort decisions in the contest stage. Section 5 analyzes the equilibrium information acquisition decisions in the game and Section 6 discusses the social optimum. Section 7 concludes the paper. The proofs are relegated to the Appendix.

2. Literature Review

There is a large literature on Tullock contests with complete information. To name a few, Baye and Hoppe (2003) identify several economic settings that are strategically equivalent to a Tullock contest, such as rent-seeking, patent races, or innovation tournaments. Pérez-Castrillo and Verdier (1992), Szidarovszky and Okuguchi (1997), and Yamazaki (2008) study the existence and uniqueness of equilibrium in rent-seeking games.[1] By contrast, the literature that considers contests with incomplete information is relatively sparse. Fey (2008) considers rent-seeking contests where players have private information about their effort cost. Wasser (2013) also studies a Tullock contest where players can be either completely informed or privately informed about their own costs. Hurley and Shogren (1998) and Malueg and Yates (2004) study the case where the players' valuations of the prize are private information. Einy et al. (2015) show that under some standard assumptions, Tullock contests with asymmetric information have pure-strategy Bayesian Nash equilibria. None of the above literature considers the possibility of using delegation in the contest.

Recently, there is a growing literature that incorporates delegation in rent-seeking contests. Baik and Kim (1997) consider the situation where the agent's ability is higher than the principal's, in the sense that his effort has a larger effect on the probability of winning the prize. They argue that the

1 See Konrad (2008) for a general survey.

principal with a higher valuation will delegate to the agent with more ability. Wärneryd (2000) shows that not delegating to an agent would be a dominant strategy for each principal, but both of them will be benefited if they both delegate, i.e., a prisoners' dilemma occurs. Schoonbeek (2002) shows that one-sided delegation occurs if the two players have different risk-attitudes. In equilibrium, a risk-averse player may hire a risk-neutral agent. The above literature deals with the situation of complete information.

The literature that considers delegation in contests when players have incomplete information about the prize value is still scarce. Like our paper, Schoonbeek (2017) considers a two-player contest for a prize with a common value, which is unknown to the principals. A principal can either delegate to an agent to act on her behalf, or enter the contest on her own. The agent can observe the true value of the prize. Under the circumstance as we consider in this paper where players have an equal prior belief over the prize value, he finds that only no delegation (i.e., neither principal delegates) and unilateral delegation (i.e., only one principal delegates) can occur in equilibrium. We also obtain a similar result that bilateral delegation (where both principals delegate) cannot constitute an equilibrium.[2] Differing from Schoonbeek (2017), however, we incorporate the option of self-acquisition of information for each principal to choose besides delegation. There emerge additional equilibria that at least one principals acquire information via self-investigation, and that one principal delegates but the other one investigates by herself, which do not appear in Schoonbeek (2017). In particular, self-investigation is better than delegation when the principal can acquire information at a small cost. The equilibria obtained in this paper are richer and more interesting than those in the literature, and they can also capture and explain the real-world situation more properly.

Another closely related paper is Morath and Münster (2013). Although they do not consider Tullock-style contests *per se*, they also study players' information acquisition investment ahead of conflicts. They assume that

2 Schoonbeek (2017) considers more general distributions where the prior belief over the high-value and low-value prize can be uneven. Thus, bilateral delegation can occur in equilibrium in some range of the parameters.

initially no player is informed about the true value of the prize, and players can acquire information at a cost to observe the true value. When a player's investment in information can be observed by the other players, they obtain a similar result like ours: in equilibrium, (i) both principals acquire information if the cost of information is sufficiently low, (ii) exactly one principal acquires information when the cost is intermediate, and (iii) both principals acquire no information if the cost is sufficiently high. Nevertheless, they argue that the equilibrium information acquisition is excessive compared to the social optimum. This is in contrast to our finding that there can be under-investment in information acquisition in the equilibrium. The key difference is that they do not consider the option of delegation. When delegation is an option, we show that it is social optimal that both principals delegate to the agents to acquire information.

There is another literature that emphasizes that delegation can be a way to utilize the agent's expertise or to provide incentives for acquiring information. Aghion and Tirole (1997) highlight the key trade-off between the loss of the agent's information acquisition incentives and the loss of her control under delegation. Szalay (2005) argues that partial delegation, in that some intermediate actions are eliminated, can force the agent to try harder in finding the best choice for the principal and then improves information collection. It has been also argued that hiring an agent with a different interest can motivate information acquisition. Dewatripont and Tirole (1999) argue that advocacy systems where one agent collects information on the cons and another one collects information on the pros can help the agents to collect information. Gerardi and Yariv (2008) also argue that it can be optimal for the principal to hire experts with different preferences, because their incentives to collect information are the strongest. Li (2001) suggests that conservatism, where the decision is made against the alternative favored by the group's preference when evidence supports it, can increase the incentives to gather evidence. Omiya et al. (2017) find that, under the principal's authority, the optimal effort level in acquiring information is at its highest when the agent has an extreme bias, while it is also at its highest when the agent has an intermediate bias under the agent's authority. Thus, the principal should keep the authority and communicate with the agent when the agent is relatively biased, and delegate the authority when the agent has an intermediate bias.

Our contribution to the literature is twofold. First, although some papers consider delegation in contests with incomplete information, they typically assumed that delegation is the only way to obtain information, and they neglected the fact that people may sometimes acquire information by themselves. Therefore, to fill the gap, we incorporate both options into the model, which makes this paper the first that incorporates both methods in the contest theory literature. We can therefore obtain richer equilibria that capture the real-world situation more properly. Second, in another large literature discussing the allocation of authority within organizations, researchers seldom consider how competition outside organizations affects the optimal allocation. We try to build up a model that not only can analyze how principals allocate the decision-making right when facing other competitors, but also combine these two strands of literature in a meaningful way.

3. The Model

We consider a Tullock-style contest where there are two teams $i, j \in \{1, 2\}$, $i \neq j$, each of which is composed of a principal and an agent and competes with each other to win a prize. The value of the prize has two possibilities, $v_\theta \in \{v_L, v_H\}$. The true state of v_θ is unknown to the principal initially, and the prior belief is that $\theta = H$ and $\theta = L$ occur with equal probability $1/2$.[3] On the contrary, the agent has the full information regarding the true state. Such information is valuable to the principal because she wishes to make the right decision, so that more effort is exerted in pursuing a prize with a high value, and less effort in pursuing a low-value one. There are two ways for the principals to obtain that information: "self-investigation," where she acquires the information and enters the contest by herself, and "delegation," where

[3] It is possible to consider a general distribution where v_H occurs with probability $q \in (0, 1)$. However, since both q and v_H represent the "benefit of the high-value prize," and $1 - q$ and v_L represent the "benefit of the low-value prize," the effect of q can be captured by the "gap in the prize values" (i.e., v_H / v_L) to some extent, as will be discussed later.

she delegates the right to an agent, who enters the contest on her behalf. We allow each principal to choose one of the following strategies: (i) acquiring no information, (ii) acquiring information via self-investigation, and (iii) acquiring information via delegation, which are specified as follows.

Principal i can choose $a_i \in \{N, S, D\}$. When $a_i = N$, i.e., the action of "no-acquisition," Principal i acquires no information and enters the contest by herself. In this case, she applies the prior belief to evaluate the value of the prize. When $a_i = S$, i.e., the action of "self-investigation," she acquires information and enters the contest by herself. For simplicity, we assume that, in such a case, she can obtain full information regarding the true state by paying a cost $k > 0$. This k can be a measure of the difficulty of being fully informed. When $a_i = D$, i.e., the action of "delegation," she delegates the right of competing in the contest to an agent, who can observe the true state θ. However, in order to induce the agent to put forth effort, the principal needs to reward him if he wins the prize. Thus, a contract that specifies $\delta_i \geq 0$ is offered to the agent, where he can obtain a share δ_i of the prize when he wins the contest, and 0 if he fails.[4] After the contract is offered, Agent i enters the contest on the principal's behalf.

We assume that a_i can be observed by the other player, so the accuracy of each player's information is a common knowledge. This assumption deserves more explanation. There is a literature that discusses information acquisition in different forms: "overt" information acquisition, which means that the decision maker can observe the quality of information acquired by the other players but not its content (such as the signal that is received), and "covert" information acquisition, where neither the quality nor the content is observable.[5] Our model belongs to the former category. A real-world example would be oil-tract auctions. Suppose that an oil tract is to be developed, and several oil companies are interested in exploring the ground and compete for the right to develop the resource. Before entering the auction, they have to acquire information about the value of the resource. To acquire such

4 That is, we assume that the agent is protected by limited liability so that he will not be punished if he loses the contest.

5 See, for example, Argenziano et al. (2016), and Morath and Münster (2013).

information, an oil company needs permissions, purchases the equipment, or hires contract consulting services before starting a drill. It incurs some costs of information acquisition through exploratory drills which are easily visible by other companies.

After entering the contest, the contestants put forth efforts to win the prize, which can be varied depending on the information that they have. If it is Principal i who enters the contest, then she makes effort e_i, while if it is Agent i who competes in the contest, he makes effort x_i. The efforts are simultaneously chosen by the contestants from Team i and Team j. As in the typical Tullock-style contest, the probability of a contestant winning the prize depends on the relative strength of effort levels put forth by the contestants.

The timing of the game proceeds as follows:

Stage 1: Both principals choose simultaneously one of the following three options: $a_i \in \{N, S, D\}$. When $a_i = N$ or S, Principal i enters the contest by herself. When $a_i = D$, Principal i delegates the contest right to Agent i and offers him a contract δ_i. Agent i then decides whether to accept the contract or not, and if he accepts the contract, he enters the contest and compete on her behalf.

Stage 2: Players who enter the contest put forth effort simultaneously.

Stage 3: The outcome of the contest and the players’ payoffs are realized.

The following remark is concerned with the timing of the game:

We assume that the agent learns the true value after the contract is offered. However, our result is robust to the alternative timing where the agent observes the true state before contracting, as long as the contract is contingent only on the outcome in the contest (i.e., a success or a failure). The reasoning is as follows. If the principal offers contracts with two different sharing rules, δ_{iH} and δ_{iL}, where $\delta_{iH} \neq \delta_{iL}$, to screen the agent's information, it requires that the L-type agent (who observes $\theta = L$) has no incentive to pretend to be the H-type one (who observes $\theta = H$), and vice versa. However, both types would end up choosing the same contract so that the principal cannot distinguish between them. Since separation through agent’s self-selection is not possible, it is equivalent to the case where the principal offers a contract with the same δ_i. We provide a proof of this Remark in the Appendix.

We now proceed with the analysis. In Stage 3, given the information acquisition and effort decisions made by the players in the previous stages, we denote by $u_{i\theta}$ and $\pi_{i\theta}$ Principal i's and Agent i's payoff obtained under the realized state $\theta \in \{H, L\}$, respectively. Then Principal i obtains:

$$u_{i\theta} = \begin{cases} \left(\dfrac{e_i}{e_1 + e_2}\right) v_\theta - e_i - \mathbf{1}(a_i) \cdot k, & \text{when } a_i,\ a_j \in \{N, S\}; \\ \left(\dfrac{e_i}{e_i + x_j}\right) v_\theta - e_i - \mathbf{1}(a_i) \cdot k, & \text{when } a_i \in \{N, S\} \text{ and } a_j = D; \\ \left(\dfrac{x_i}{x_i + e_j}\right) (1 - \delta_i) v_\theta, & \text{when } a_i = D, \text{ and } a_j \in \{N, S\}; \\ \left(\dfrac{x_i}{x_1 + x_2}\right) (1 - \delta_i) v_\theta, & \text{when } a_1 = a_2 = D, \end{cases} \tag{1}$$

where $i, j \in \{1, 2\}$ and $i \neq j$. $\mathbf{1}(a_i)$ is an indicator function, where $\mathbf{1}(S) = 1$ and $\mathbf{1}(N) = 0$.

For example, when both principals enter the contest by themselves (i.e., $a_i, a_j \in \{N, S\}$) and have made efforts e_1 and e_2, the probability of Principal i winning the contest is $e_i / (e_1 + e_2)$. When her opponent j delegates while she does not (i.e., $a_i \in \{N, S\}$ and $a_j = D$), Principal i enters the contest by herself and wins with probability $e_i / (e_i + x_j)$. Moreover, Principal i needs to pay an acquisition cost k if $a_i = S$. On the other hand, when Principal i delegates while her opponent does not (i.e., $a_i = D$ and $a_j \in \{N, S\}$), Agent i competes with Principal j and wins the contest with probability $x_i / (x_i + e_j)$; however, when the team wins, Principal i has to give up $\delta_i v_\theta$ to the agent. Finally, when both principals delegate (i.e., $a_1 = a_2 = D$), then both agents compete with each other in the contest, and Principal i wins with probability $x_i / (x_i + x_j)$ and pays $\delta_i v_\theta$ to the agent when the team wins.

Agent i's payoff when he is delegated by Principal i under the realized state θ is

$$\pi_{i\theta} = \begin{cases} \left(\dfrac{x_i}{x_i + e_j}\right)\delta_i v_\theta - x_i, & \text{when } a_i = D \text{ and } a_j \in \{N, S\}; \\ \left(\dfrac{x_i}{x_1 + x_2}\right)\delta_i v_\theta - x_i, & \text{when } a_1 = a_2 = D; \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

4. The Optimal Effort Decisions in the Contest

In this section, we analyze the players' optimal decisions in the contest in Stage 2, given the principals' information acquisition decisions made in Stage 1. We separate the six scenarios in four categories:

4.1 Acquiring no Information

Scenario 1: Neither principal self-investigates or delegates, i.e., $(a_1, a_2) = (N, N)$.

In this case, both principals remain unknown about the true state, i.e., each principal believes that $v_\theta = v_H$ with probability 1 / 2 and knows that her opponent also has the same quality of information. Then according to (1), Principal 1 enters the contest and chooses the effort level e_1 (which cannot be contingent on the true state) to solve the following problem:

$$\max_{e_1} \left(\frac{e_1}{e_1 + e_2}\right)\left(\frac{1}{2}v_H + \frac{1}{2}v_L\right) - e_1.$$

The first-order condition is $e_2 / (e_1 + e_2)^2 [v_H / 2 + v_L / 2] = 1$, which gives us Principal 1's best response function:

$$e_1(e_2) = \sqrt{\left(\frac{1}{2}v_H + \frac{1}{2}v_L\right)e_2} - e_2. \tag{3}$$

The situation for Principal 2 is analogous. By solving their best response functions, we find that the equilibrium effort levels in the subgame of Stage 2 are

$$e_1^* = e_2^* = \frac{v_H + v_L}{8}. \tag{4}$$

We denote by p_i the probability that Principal i wins the contest. In this case,

$$p_1^* = p_2^* = \frac{1}{2}. \tag{5}$$

It follows that the principals' expected payoffs obtained in the contest are

$$u_1^* = u_2^* = \frac{v_H + v_L}{8}. \tag{6}$$

It is easy to see that because the principals face the symmetric problem when they both acquire full information in the same way, the winning probability and expected payoff are equal.

4.2 Acquiring Information via Self-investigation

The next two cases consider the situations where at least one principal acquires information via self-investigation.

Scenario 2: Both principals acquire information via self-investigation, i.e., $(a_1, a_2) = (S, S)$.

In this case, both principals fully learn the true state, and they also know that their opponent has the same information. Then Principal i chooses the effort level $e_{i\theta}$ which is contingent on the true state $\theta \in \{H, L\}$ by solving the following problem, given the effort level $e_{j\theta}$ chosen by her opponent j:

$$\max_{e_{i\theta}} \left(\frac{e_{i\theta}}{e_{1\theta} + e_{2\theta}} \right) v_\theta - e_{i\theta} - k.$$

Principal i's best response function when observing θ is $e_{i\theta} = \sqrt{v_\theta e_{j\theta}} - e_{j\theta}$. It is easy to check that the equilibrium effort levels are

$$e_{1\theta}^* = e_{2\theta}^* = \frac{v_\theta}{4}. \tag{7}$$

In this case, each principal wins the contest with equal probability:

$$p_{1\theta}^* = p_{2\theta}^* = \frac{1}{2}, \tag{8}$$

and the expected payoffs for the principals is $u_{i\theta} = v_\theta / 4 - k$. It follows that the principals' *ex ante* payoffs (i.e., before entering the contest and observing the true state) earned in the contest are

$$u_1^* = \frac{1}{2}u_{1H} + \frac{1}{2}u_{1L} = \frac{v_H + v_L}{8} - k = u_2^*. \tag{9}$$

By comparing Scenario 1 and Scenario 2 , we see an interesting feature in the Tullock-style contest game: acquiring more information does not necessarily make the principal better off in the *ex ante* sense. When both principals acquire information, even though they can observe the true state, they exert the same effort level on average as that when they both acquire no information. Thus, when the opponent also acquires the information, having more information does not give the advantage in the winning probability in the contest but an extra cost needs to be paid.

Scenario 3: Only one principal acquires information via self-investigation, i.e., $(a_1, a_2) = (S, N)$ or (N, S).

Consider the situation where $(a_1, a_2) = (S, N)$. The other case is analogous. After entering the contest, Principal 1 has full information, while Principal 2 remains unaware of the true state. They both know each other's quality of information. Principal 1 then chooses the effort level $e_{1\theta}$ which can be contingent on the true state, while Principal 2 can only choose one effort level e_2 which does not depend on the true state. Thus, Principal 1 chooses the effort level $e_{1\theta}$ to solve the following problem:

$$\max_{e_{1\theta}} \left(\frac{e_{1\theta}}{e_{1\theta} + e_2} \right) v_\theta - e_{1\theta} - k.$$

Principal 1's best response function is $e_{1\theta}(e_2) = \sqrt{v_\theta e_2} - e_2$.

On the other hand, since she in unaware of the true state, Principal 2 chooses the effort level e_2 to solve the following problem:

$$\max_{e_2} \frac{1}{2} \left(\frac{e_2}{e_{1H} + e_2} \right) v_H + \frac{1}{2} \left(\frac{e_2}{e_{1L} + e_2} \right) v_L - e_2.$$

The first-order condition satisfies $v_H e_{1H} / 2(e_{1H} + e_2)^2 + v_L e_{1L} / 2(e_{1L} + e_2)^2 = 1$. We can find the equilibrium effort levels as follows:

$$e_{1H}^* = \frac{3v_H - v_L + 2\sqrt{v_H v_L}}{16}, \quad e_{1L}^* = \frac{3v_L - v_H + 2\sqrt{v_H v_L}}{16}, \quad e_2^* = \frac{(\sqrt{v_H} + \sqrt{v_L})^2}{16}. \tag{10}$$

In this case, the probabilities that Principal 1 wins the contest are

$$p_{1H}^* = \frac{e_{1H}^*}{e_{1H}^* + e_2^*} = \frac{3}{4} - \frac{\sqrt{v_L}}{4\sqrt{v_H}}, \quad p_{1L}^* = \frac{e_{1L}^*}{e_{1H}^* + e_2^*} = \frac{3}{4} - \frac{\sqrt{v_H}}{4\sqrt{v_L}}. \tag{11}$$

There are two subcases to consider:

Case (a): $v_H < 9v_L$.

In this case, the gap in the prize value is relatively small, and the effort levels in (10) and the probabilities in (11) are all positive. Therefore, Principal 1's payoffs conditional on the true state are

$$u_{1H}^* = p_{1H}^* v_H - e_{1H}^* - k = \frac{9v_H}{16} + \frac{v_L}{16} - \frac{3\sqrt{v_H v_L}}{8} - k,$$
$$u_{1L}^* = p_{1L}^* v_L - e_{1L}^* - k = \frac{v_H}{16} + \frac{9v_L}{16} - \frac{3\sqrt{v_H v_L}}{8} - k. \tag{12}$$

Thus, the *ex ante* payoff for Principal 1 in the contest stage is

$$u_1^* = \frac{5(v_H + v_L)}{16} - \frac{3\sqrt{v_H v_L}}{8} - k. \tag{13}$$

On the other hand, Principal 2's *ex ante* payoff in this stage is

$$u_2^* = \frac{1}{2}(1 - p_{1H}^*) v_H + \frac{1}{2}(1 - p_{1L}^*) v_H - e_2 = \frac{(\sqrt{v_H} + \sqrt{v_L})^2}{16}. \tag{14}$$

Case (b): $v_H \geq 9v_L$.

According to (10) and (11), $e_{1L}^* \leq 0$ and $p_{1L}^* \leq 0$ when $v_H \geq 9v_L$. That is, when the gap in the prize value is relatively large, the effort level and winning probability when $\theta = L$ should be set to zero. Intuitively, when the value of low prize is too small, Principal 1 will give up making effort when she finds out that the prize value is too low. Principal 2 also anticipates this and will choose another effort level. Principal 1's best response function when $\theta = H$ remains $e_{1H}(e_2) = \sqrt{v_H e_2} - e_2$, and Principal 2 chooses the effort level e_2 to solve the following problem:

$$\max_{e_2} \frac{1}{2}\left(\frac{e_2}{e_{1H} + e_2}\right) v_H + \frac{1}{2} v_L - e_2,$$

which means that Principal 2 will win the low prize for sure because only she makes effort when $\theta = L$. Her best response function satisfies $e_2(e_{1H}) = \sqrt{v_H e_{1H} / 2} - e_{1H}$. We then can find the equilibrium effort levels and the winning probabilities as follows:

$$e_{1H}^{*}=\frac{2v_H}{9},\quad e_{1L}^{*}=0,\quad e_2^{*}=\frac{v_H}{9},\quad p_{1H}^{*}=\frac{2}{3},\quad p_{1L}^{*}=0. \tag{15}$$

The principals' *ex ante* payoff in the contest stage are:

$$u_1^{*}=\frac{2v_H}{9}-k,\ \text{and}\ u_2^{*}=\frac{v_H}{18}+\frac{v_L}{2}. \tag{16}$$

By observing (10) and (15), we can see that when the gap in the prize value becomes larger, the principals put forth effort only to pursue the high-value prize. Therefore, the principals' incentives to make effort will be higher in chasing the high value (i.e., $2v_H/9\geq(3v_H-v_L+2\sqrt{v_Hv_L})/16$ when $v_H\geq 9v_L$).

4.3 Acquiring Information via Delegation

This subsection considers the situation when at least one principal chooses to delegate. When delegation takes place, the agent has the precise information and so it is beneficial for the principal to delegate; however, in order to induce the agent to make effort in winning the contest, the principal needs to offer him the incentives. Therefore, the trade-off is between the benefit of having precise information and the cost of inducing the agent's effort.

Scenario 4: Both principals acquire information via delegation, i.e., $(a_1, a_2) = (D, D)$.

Given the contracts offered by the principals, δ_1 and δ_2, when Agent i observes $\theta\in\{H, L\}$, he chooses effort level $x_{i\theta}$ to solve the following problem in the contest stage:

$$\max_{x_{i\theta}}\left(\frac{x_{i\theta}}{x_{1\theta}+x_{2\theta}}\right)\delta_1 v_\theta - x_{i\theta}.$$

The agents' equilibrium effort levels given the contracts offered by the principals, (δ_1, δ_2), are

$$x_{1\theta}^{*}(\delta_1,\delta_2)=\frac{\delta_1^2\delta_2}{(\delta_1+\delta_2)^2}v_\theta,\quad x_{2\theta}^{*}(\delta_1,\delta_2)=\frac{\delta_1\delta_2^2}{(\delta_1+\delta_2)^2}v_\theta. \tag{17}$$

In this case, Principal i wins the contest with probability $p_{i\theta}=\delta_i/(\delta_1+\delta_2)$. She then chooses a δ_i to solve

$$\max_{\delta_i}(1-\delta_i)\left(\frac{\delta_i}{\delta_1+\delta_2}\right)\left(\frac{1}{2}v_H+\frac{1}{2}v_L\right).$$

We find that in equilibrium, the optimal shares of prize that Agent 1 and Agent 2 receive are

$$\delta_1^*=\delta_2^*=\frac{1}{3}. \tag{18}$$

It follows that the equilibrium effort levels made by the agents and the winning probability are

$$x_{1\theta}^*=x_{2\theta}^*=\frac{v_\theta}{12},\quad p_{1\theta}^*=\frac{1}{2}. \tag{19}$$

The principals' *ex ante* payoffs obtained from the contest stage are

$$u_1^*=u_2^*=\frac{v_H+v_L}{6}. \tag{20}$$

Compared to the situation where the principal investigates by herself, the agent has less of an incentive to exert effort because he only receives a part of the prize when he wins. Therefore, the equilibrium effort levels are much lower than those when the principals enter the contest by themselves. Moreover, when both delegate, their quality of information remains equal, and so their winning probabilities are also the same and no one has the advantage in the contest.

Scenarios 5: Only one principal acquires information via delegation, i.e., $(a_1, a_2) = (D, N)$ or (D, N).

Consider the situation where Principal 1 delegates while Principal 2 does not acquire information via delegation or self-investigation, i.e., $(a_1, a_2) = (D, N)$, and so Agent 1 and Principal 2 will compete with each other in the contest stage. Given the contract offered by Principal 1, δ_1, when Agent 1 observes the true state θ, he chooses effort level $x_{1\theta}$ to solve the following problem:

$$\max_{x_{1\theta}} \left(\frac{x_{1\theta}}{x_{1\theta} + e_2} \right) \delta_1 v_\theta - x_{1\theta}.$$

On the other hand, Principal 2 has no accurate information and thus has to choose the effort level e_2 to solve the following problem:

$$\max_{e_2} \frac{1}{2} \left(\frac{e_2}{x_{1H} + e_2} \right) v_H + \frac{1}{2} \left(\frac{e_2}{x_{1L} + e_2} \right) v_L - e_2.$$

The equilibrium effort levels and Principal 1's winning probability in this subgame, given the contract δ_1, are

$$\begin{aligned} x_{1\theta}^* &= \frac{\delta_1(\sqrt{v_H} + \sqrt{v_L})}{4(1+\delta_1)^2} \left[2\sqrt{v_\theta}(1+\delta_1) - (\sqrt{v_H} + \sqrt{v_L}) \right], \\ e_2^* &= \frac{\delta_1(\sqrt{v_H} + \sqrt{v_L})^2}{4(1+\delta_1)^2}, \quad p_{1\theta}^* = 1 - \frac{\sqrt{v_H} + \sqrt{v_L}}{2(1+\delta_1)\sqrt{v_\theta}}. \end{aligned} \tag{21}$$

Similar to the previous Scenario 3, it is possible that when the gap in the prize value is relatively large, the agent may not want to make any effort when he realizes that he is pursuing a low prize. We separate two subcases:

Case (a): $v_H < 3v_L$.

Principal 1 chooses a δ_1 to solve

$$\max_{\delta_1} \frac{1}{2}\left[1-\frac{\sqrt{v_H}+\sqrt{v_L}}{2(1+\delta_1)\sqrt{v_H}}\right](1-\delta_1)v_H + \frac{1}{2}\left[1-\frac{\sqrt{v_H}+\sqrt{v_L}}{2(1+\delta_1)\sqrt{v_L}}\right](1-\delta_1)v_L.$$

We find that in equilibrium:

$$\delta_1^* = \frac{\sqrt{v_H}+\sqrt{v_L}}{\sqrt{v_H+v_L}} - 1. \tag{22}$$

By substituting (22) into (21), we find that $x_{1\theta}^* > 0$ and $p_{1\theta}^* > 0$ as long as $2\sqrt{v_L} - \sqrt{v_H+v_L} > 0$ or $v_H < 3v_L$, where

$$\begin{aligned} x_{1\theta}^* &= \frac{1}{4}(\sqrt{v_H}+\sqrt{v_L}-\sqrt{v_H+v_L})(2\sqrt{v_\theta}-\sqrt{v_H+v_L}), \\ e_2^* &= \frac{1}{4}(\sqrt{v_H}+\sqrt{v_L}-\sqrt{v_H+v_L})\sqrt{v_H+v_L}, \quad p_{1\theta}^* = 1-\frac{\sqrt{v_H+v_L}}{2\sqrt{v_\theta}}. \end{aligned} \tag{23}$$

The principals' *ex ante* payoffs in the contest stage can be found accordingly:

$$u_1^* = \frac{[2\sqrt{v_H+v_L}-(\sqrt{v_H}+\sqrt{v_L})]^2}{4}, \text{ and } u_2^* = \frac{v_H+v_L}{4}. \tag{24}$$

Case (b): $v_H \geq 3v_L$.

Note that in (23), $x_{1L}^* \leq 0$ and $p_{1L}^* \leq 0$ when $v_H \geq 3v_L$. In this case, the effort level and winning probability when $\theta = L$ should be set to zero, that is, Agent 1 will give up making effort when she observes $\theta = L$ because it is not worth it. Principal 2 anticipates this and will choose an effort level different from that in the above Case (a).

Agent 1's decision again follows (21) when observing $\theta = H$. Principal 2 chooses the effort level e_2 to solve the following problem:

$$\max_{e_2} \frac{1}{2}\left(\frac{e_2}{x_{1H}+e_2}\right)v_H + \frac{1}{2}v_L - e_2,$$

which means that Principal 2 will definitely win the low prize because she is the only one who makes effort when $\theta = L$. Her best response function satisfies $e_2(x_{1H}) = \sqrt{v_H x_{1H}/2} - e_{1H}$. Then the equilibrium effort levels in the contest stage and Principal 1's winning probability are

$$x_{1H}^* = \frac{2(\delta_1)^2 v_H}{(1+2\delta_1)^2}, \quad x_{1L}^* = 0, \quad e_2^* = \frac{\delta_1 v_H}{(1+2\delta_1)^2}, \quad p_{1H}^* = \frac{2\delta_1}{1+2\delta_1}, \quad p_{1L}^* = 0. \tag{25}$$

Principal 1 then chooses a δ_1 to solve

$$\max_{\delta_1} \frac{1}{2}\left(\frac{2\delta_1}{1+2\delta_1}\right)(1-\delta_1)v_H.$$

We find that in equilibrium:

$$\delta_1^* = \frac{\sqrt{3}-1}{2}. \tag{26}$$

By substituting (26) into (21), we find that

$$x_{1H}^* = \frac{(\sqrt{3}-1)^2 v_H}{6}, \quad x_{1L}^* = 0, \quad e_2^* = \frac{(\sqrt{3}-1)v_H}{6}, \quad p_{1H}^* = \frac{\sqrt{3}-1}{\sqrt{3}}, \quad p_{1L}^* = 0. \tag{27}$$

The principals' *ex ante* payoffs in the contest stage can be found accordingly:

$$u_1^* = \frac{(\sqrt{3}-1)^2 v_H}{4} \text{ and } u_2^* = \frac{v_H}{6} + \frac{v_L}{2}. \tag{28}$$

4.4 Acquiring Information via Different Methods

The last situation is when both principals acquire information but in different ways: one principal acquires information via self-investigation, while the other one acquires information via delegation.

Scenarios 6: One principal investigates by herself while the other delegates, i.e., $(a_1, a_2) = (S, D)$ or (D, S).

Consider the situation where Principal 1 investigates by herself while Principal 2 delegates, i.e., $(a_1, a_2) = (S, D)$. Thus, Principal 1 and Agent 2 will compete with each other in the contest stage, and both are fully informed about the true state θ. Given the contract offered by Principal 2, δ_2, Principal 1 chooses effort level $e_{1\theta}$ to solve the following problem:

$$\max_{e_{1\theta}} \left(\frac{e_{1\theta}}{e_{1\theta} + x_2} \right) v_\theta - e_{1\theta} - k.$$

On the other hand, Agent 2 chooses the effort level x_2 to solve the following problem:

$$\max_{x_{2\theta}} \left(\frac{x_{2\theta}}{e_{1\theta} + x_2} \right) \delta_2 v_\theta - x_{2\theta}.$$

The chosen effort levels and Principal 1's winning probability in this subgame, given the contract δ_2, are

$$e_{1\theta}^* = \frac{\delta_2 v_\theta}{(1+\delta_1)^2}, \quad x_{2\theta}^* = \frac{(\delta_2)^2 v_\theta}{(1+\delta_1)^2}, \quad p_{1\theta}^* = \frac{1}{1+\delta_2}. \tag{29}$$

Given (29), Principal 2 chooses a δ_2 to solve

$$\max_{\delta_2} \frac{1}{2} \left(\frac{\delta_2}{1+\delta_2} \right) (1-\delta_2)(v_H + v_L).$$

We find that in equilibrium:

$$\delta_2^* = \sqrt{2} - 1. \tag{30}$$

By substituting (30) into (29), we find that

$$e_{1\theta}^* = \frac{(\sqrt{2}-1)v_\theta}{2}, \quad x_{2\theta}^* = \frac{(\sqrt{2}-1)^2 v_\theta}{2}, \quad p_{1H}^* = p_{1L}^* = \frac{1}{\sqrt{2}}. \tag{31}$$

The principals' *ex ante* payoffs in the contest stage can be found accordingly:

$$u_1^* = \frac{v_H + v_L}{4} - k, \quad u_2^* = \frac{(\sqrt{2}-1)^2(v_H + v_L)}{2}. \tag{32}$$

By comparing the results in the last three scenarios, we derive an interesting implication regarding the incentives that a principal needs to give to the agent under delegation:

Proposition 1.
Consider the scenarios where Principal i acquires information via delegation. Then the share of prize needed to be given to the agent, δ_i^*, is the highest when her opponent j acquires information via self-investigation, and the lowest when j also delegates.

Proof. According to (18), (22), (26) and (30), the comparison is immediate: $\sqrt{2} - 1 > (\sqrt{v_H} + \sqrt{v_L}) / \sqrt{v_H + v_L} - 1 > 1/3$ if $v_H < 3v_L$, and $\sqrt{2} - 1 > (\sqrt{3} - 1)/2 > 1/3$ if $v_H \geq 3v_L$.

□

The intuition for this result is the following. As explained before, when a principal delegates, the agent has less of an incentive to exert effort compared to the situation where the principal investigates by herself. Thus, the incentives needed to induce the agent's effort is the lowest among all the three cases when the opponent principal also delegates because the opponent agent makes less effort as well. In the other two scenarios where the opponent principal enters the contest by herself, a stronger incentive needed to be offered when the opponent acquires information, because she also makes more effort when she has information.

4.5 Who has the Advantage in Winning the Contest?

Based on the above analysis, we can compare the *ex ante* (i.e., before entering the contest and observing the true state) winning probability in the contest stage in the equilibrium among different scenarios. The *ex ante* winning probability for Principal i is defined as $1/2p_{iH}^{*}+1/2p_{iL}^{*}$ if she acquires information via either self-investigation or delegation, or simply p_i^* if she does not acquire information. This probability is determined by the effort levels made by both contestants and therefore can measure their relative advantage in winning the contest. When a principal gains an expected winning probability larger than $1/2$, she is considered having an advantage in winning the contest. The question is: who can have the advantage in each scenario?

We have the following finding:

Proposition 2.
Given the information acquisition decisions in Stage 1, in the contest stage, where $i, j \in 1, 2$, $i \neq j$*:*

(1) If both principals adopt the same method in acquiring information, i.e., $(a_i, a_j) = (S, S)$ *or (D, D), or neither one acquires information, i.e.,* $(a_i, a_j) = (N, N)$*, then their ex ante winning probabilities are equal.*

(2) If only one principal acquires information, i.e., $(a_i, a_j) = (S, N)$ *or (D, N), then the one who acquires no information has the advantage in winning the contest.*

(3) If the principals adopt different methods in acquiring information, i.e., $(a_i, a_j) = (S, D)$*, then the one who acquires information via self-investigation has the advantage in winning the contest.*

Proof.

(1) When $a_1 = a_2$, then according to (5) in Scenario 1, (8) in Scenario 2 and (19) in Scenario 4, the *ex ante* probabilities in these scenarios are all equal to $1/2$ for both principals.

(2) When $(a_i, a_j) = (S, N)$ or (D, N), from (11) and (15) in Scenario 3, for the principal who acquires information via self-investigation, her winning probabilities are $1/2(3/4 - \sqrt{v_L}/4\sqrt{v_H}) + 1/2(3/4 - \sqrt{v_H}/4\sqrt{v_L}) = 3/4 - 1/8(\sqrt{v_L}/\sqrt{v_H} + \sqrt{v_H}/4\sqrt{v_L}) < 1/2$ and $(1/2)(2/3) = 1/3 < 1/2$, and from (23) and (27) in Scenario 4, for the principal who acquires information via delegation, her winning probabilities are $1/2(1 - \sqrt{v_H + v_L}/2\sqrt{v_H}) + 1/2(1 - \sqrt{v_H + v_L}/2\sqrt{v_L}) = 1 - (\sqrt{v_H + v_L})(\sqrt{v_H + v_L})/4\sqrt{v_H v_L} < 1/2$ and $1/2 \cdot \sqrt{3} - 1/\sqrt{3} < 1/2$. Both results mean that the one who acquires no information has the advantage in winning the contest in the *ex ante* sense.

(3) When $(a_i, a_j) = (S, D)$ as discussed in Scenario 6, then from (31), $1/\sqrt{2} > 1/2$., which means that the *ex ante* probability is larger for the principal who acquires information via self-investigation and thus she has the advantage.

□

This result highlights an important feature of contests with a common value under incomplete information. When both principals acquire information, although the precise information helps them to correctly make the right decision in effort so that there is no mismatch (i.e., making a lot of efforts in chasing a low-value prize, or vice versa), this does not improve their position in winning the contest. Their effort levels and the winning probabilities in each state remain equal whereas they each need to pay the cost of acquiring the information. Thus, in the *ex ante* sense, having both principal acquire full information does not give the advantage to either principal.

On the other hand, if only one of them acquires information through self-investigation, surprisingly, she is at a disadvantage in winning the contest compared to her opponent in the *ex ante* sense. Since she can make the effort decision more efficiently with precise information, it is straightforward that she will make more effort in pursuing the high prize, and less effort in the low prize. However, on average, the effort level is lower than that when both have the same quality of information. Reacting to this, the one with no information put forth more effort in the contest, so that her winning probability will be higher than her opponent.

The driving force under delegation is different. When only one principal delegates, even though her information is now precise, she needs to offer a proportion of the prize to the agent. Compared to her own authority, the agent's incentive to make effort is lower because he only obtains a part of the prize. On the other hand, the principal who acquires no information reacts to this by increasing her effort even though she has no information. Thus, the winning probability for the principal is in fact lower compared to the case of no delegation or that when both delegate.

Finally, when both acquire information by using different methods, although they both have full information, the agent's incentive of making effort under delegation is lower than when the principal enters the contest by herself. Therefore, the one who acquires information via self-investigation has the advantage in winning the contest when her opponent delegates.

5. The Equilibrium Information Acquisition Decisions

In this section, we analyze the equilibrium information acquisition decisions in Stage 1. The payoff matrices are shown in Table A1, Table A2 and Table A3 in the Appendix, where we list the ex ante payoffs for each principal obtained in Stage 2 under different strategy profiles. We will focus on the pure-strategy equilibria in the first stage, denoted by (a_1^*, a_2^*).

We have the following result:

Proposition 3.
The information acquisition decisions in the equilibrium is the following:

(1) When $v_H < 9v_L$*, then there exists an equilibrium where:*
(a) $(a_1^*, a_2^*) = (N, N)$ *when* $k \geq 3(\sqrt{v_H} - \sqrt{v_L})^2 / 16$.

(b) $(a_1^*, a_2^*) = (S, N)$ *when* $(\sqrt{v_H} - \sqrt{v_L})^2 / 16 \le k \le 3(\sqrt{v_H} - \sqrt{v_L})^2 / 16.$

(c) $(a_1^*, a_2^*) = (S, S)$ *when* $k \le (\sqrt{v_H} - \sqrt{v_L})^2 / 16.$

(2) When $v_H \ge 9v_L$*, then there exists an equilibrium where:*[6]

(a) $(a_1^*, a_2^*) = (N, N)$ *when* $v_H \le v_L / (7 - 4\sqrt{3})$ *and* $k \ge 7v_H / 72 - v_L / 8.$

(b) $(a_1^*, a_2^*) = (S, N)$ *when* $v_H \le 9(\sqrt{2} - 1)v_L / 13 - 9\sqrt{2}$ *and* $5v_H / 72 - 3v_L / 8 \le k \le 7v_H / 72 - v_L / 8.$

(c) $(a_1^*, a_2^*) = (S, S)$ *when* $k \le \min[5v_H / 72 - 3v_L / 8, (8\sqrt{2} - 11)(v_H + v_L) / 8].$

(d) $(a_1^*, a_2^*) = (D, N)$ *when* $v_H \ge v_L / (7 - 4\sqrt{3})$ *and* $k \ge (9\sqrt{3} - 14)v_H / 18.$

(e) $(a_1^*, a_2^*) = (D, S)$ *when* $v_H \ge 9(\sqrt{2} - 1)v_L / (13 - 9\sqrt{2})$ *and* $(8\sqrt{2} - 11)(v_H + v_L) / 8 \le k \le v_H / 12 - v_L / 4.$

Proof. See the Appendix.

□

We demonstrate the equilibrium information acquisition decisions in the following Figure 1. Without loss, we normalize $v \equiv v_H / v_L$. The equilibrium is characterized by the parameters v and k.

As we can see in the figure, neither principal engages in acquiring information when the gap in the prize values (measured by v) is relatively small and the cost of information acquisition (measured by k) is relatively large (represented by the regime (N, N)). One of the principals begins to acquire information via self-investigation when the gap in the prize values and the cost of information are intermediate (Regime (S, N)). When the gap in the prize values is relatively large and the cost of information is relatively small, both principals will investigate by themselves (Regime (S, S)). On the other hand, delegation by one principal can be adopted in the equilibrium only when both the gap in the prize values and the cost of information acquisition are sufficiently large (Regimes (D, N) and (S, D)). However, there does not exist a pure-strategy equilibrium where both principals delegate.

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6 We note that there is no pure-strategy equilibrium when $v_H > 9(\sqrt{2} - 1)v_L / 13 - 9\sqrt{2}$ and $v_H / 12 - v_L / 4 < k < \min[7v_H / 72 - v_L / 8, (9\sqrt{3} - 14)v_H / 18]$.

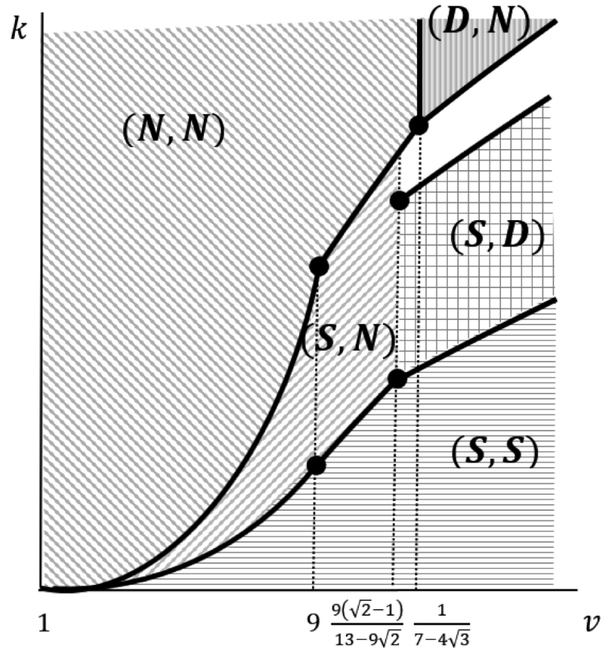

Figure 1 The Equilibrium Information Acquisition Decisions

The advantage from acquiring information is that the principal can make a more efficient effort decision by putting forth more effort in pursuing the high-value prize and less effort in pursuing the low-value prize. When the gap in the prize values is sufficiently large, the benefit from making the correct decision is also large. On the other hand, self-investigation is worth doing when the cost of information acquisition is relatively small. In such a case, both principals will find it better to acquire information via self-investigation because the benefit can cover the acquisition cost. By contrast, if the cost of information acquisition is large and the gap in the prize values is small, the benefit of information is outweighed by its cost, since players have less of an incentive to make effort in pursing the high-value prize. Thus, acquiring information is not worthwhile and so neither principal acquires information.

When both the gap in the prize values and the acquisition cost are intermediate, there is an equilibrium where one principal acquires information by herself, while the other one does not. The intuition for this result is as follows. Suppose that Principal 1 acquires information and Principal 2 does not. Given that Principal 2 does not acquire information, the benefit for Principal 1 to make a more efficient effort decision from acquiring information is larger than that if she instead chooses not to acquire information. Moreover, on average, the expected effort cost is also lower than that when both have equal quality of information. She also incurs an acquisition cost; however, if the cost k is not too large, the benefit can outweigh the cost so that it is the best response for Principal 1 to acquire information via self-investigation.

On the other hand, given that Principal 1 acquires information, the benefit for Principal 2 not to acquire information is to save the acquisition cost, at the expense of making an inefficient effort decision. As long as the acquisition cost k is not too small, the benefit can outweigh the cost, so Principal 2 would rather remain ignorant about the true state than acquire information. Combining the previous two facts, it is obvious that there exists some intermediate range of k such that only one principal acquires information in the equilibrium.

Delegation takes place in the equilibrium only when both v and k are sufficiently large. On the one hand, there is an agency cost under delegation: in order to induce the agent to make effort in the contest, the principal needs to offer a share of the prize to the agent. Compared to self-investigation, it can be more costly to induce the agent to make effort than to do it by herself. However, when the high value of the prize is large enough, the agent also has a stronger incentive to make effort, so eventually delegation can outperform self-investigation when the cost of investigation is sufficiently large. Thus, given that her opponent investigates by herself or does not acquire information, a principal would find it beneficial to choose delegation.

One particularly interesting result is that there is no equilibrium where both principals delegate. The reason is that, given her opponent delegates, principal would rather deviate to acquiring no information than delegate, because not only can she obtain a higher winning probability (as in Proposition 2), but she can also save the investigation or agency cost. Thus, it is always profitable for the principal to deviate away from delegation. This result is in marked contrast to the social optimum, as will be discussed below.

6. The Socially Optimal Decision-making

We now turn to analyzing the optimal information acquisition decision from the society's point of view, in the sense that it maximizes the total *ex ante* payoffs of all players. First of all, we compute the agent's expected payoff, who obtains a positive payoff only when he is delegated. When $(a_1, a_2) = (D, D)$, the *ex ante* payoff in the contest stage in the equilibrium for Agent i when he is delegated is

$$\pi_i^* = \frac{v_H + v_L}{24},$$

when $(a_i, a_j) = (D, N)$

$$\pi_i^* = \begin{cases} \frac{1}{4}(\sqrt{v_H} + \sqrt{v_L} - \sqrt{v_H + v_L})(3\sqrt{v_H + v_L} - 2\sqrt{v_H} - 2\sqrt{v_L}), & \textit{if } v_H < 3v_L, \\ \frac{(\sqrt{3}-1)^3 v_H}{12}, & \textit{if } v_H \geq 3v_L, \end{cases}$$

and when $(a_i, a_j) = (D, S)$,

$$\pi_i^* = \frac{(\sqrt{2}-1)^3 (v_H + v_L)}{4}.$$

The social welfare is defined as $\sum_{i=1}^{2} u_i^* + \sum_{i=1}^{2} \pi_i^*$. Based on the previous analysis, the social welfare for each strategy profile is shown in Table A4, Table A5, Table A6 in the Appendix. By comparing the social welfare under different strategy profiles, it is easy to find that $(a_1, a_2) = (D, D)$ yields the highest social welfare among all cases. We have the following result:

Proposition 4.
It is socially optimal for both principal to delegate. Therefore, from the society's point of view, the principals delegate too little in the equilibrium.

The reasoning for this result is the following. When both principals delegate, they have full information regarding the true state, so there is no loss in the making the right effort decision. Moreover, the delegated agents exert the lowest level of effort among all cases, which means that the cost "wasted" in the contest is also the lowest from the society's point of view. Thus, delegation by both principals is the social optimum. However, as argued before, this scenario can never constitute an equilibrium in the game. Hence, there is an "under-delegation" situation: the principals keeps too much authority (by entering the contest themselves) while they should have let the agents compete for them. They also under-invest in information because they do not acquire information sometimes in the equilibrium.

The result yields some policy implication. By using the example of oil tract contest, it is suggested that the oil companies should coordinate in hiring agencies who enter the contest on their behalf. Moreover, both companies should acquire information from the society's point of view. Thus, by making the information more available in the contest can increase the social welfare of the contestants. However, the situation is similar to the typical Prisoners' dilemma problem, in that players will not follow the socially optimal decision when they are self-interested.

7. Conclusion

In this paper, we consider the situation where two teams compete in a contest with a common value. There are two methods for a principal to acquire information: "self-investigation" and "delegation." We find that when the gap in the prize values is relatively large and the cost of information acquisition is relatively small, the equilibrium is such that one or both principals adopt self-investigation. On the other hand, if both the gap in the prize values and the cost of information acquisition are sufficiently large, delegation by exactly one principal can be an equilibrium. However, there is no equilibrium where both principals delegate, even though it is socially optimal for them to do so. This implies that the principals tend to delegate too little by excessively entering the contest by themselves while they should have delegated.

The current model can be extended into the following two directions. First, we may introduce other asymmetries into the model. First, the value of the prize in the contest is assumed to be the same for both teams, i.e., a common value. One may imagine another situation where the values of the prize are different for the two teams. For example, the benefits for the plaintiff and the defendant may be different when they win the lawsuit. This will give them different incentives in acquiring information and in delegating the right to contest to the agent. It is then possible to support an equilibrium where both principals delegate. Another direction is to consider the situation where a prior belief regarding the true state is different from $1/2$. In this case, the principals may already lean toward some state, and it may affect the benefit of having precise information and the relative performance of these two methods. We aim to tackle these issues in the future.

Appendix

Remark in Section 3.

Suppose that the agent learns the true state before contracting, and the principal offers contracts contingent on the outcome in the contest (i.e., a success or a failure) to screen out the agent's information. Then separation through agent's self-selection is not possible.

Proof. Consider the situation where both principals delegate. Suppose that, in the first stage, the principal offers contracts with two different sharing rules, δ_{iH} and δ_{iL}, where $\delta_{iH} \neq \delta_{iL}$, and Agent i can only accept one of them. Before accepting the contract, the agents have learned the true state $\theta = H$. In order to screen out the agent's information, it requires that the L-type agent (who knows $\theta = L$) has no incentive to pretend to be the H-type one (who knows $\theta = H$), and vice versa.

Suppose that Agent 1 chooses to accept the contract δ_{1H} (which reveals that his information indicates $\theta = H$). Then based on the analysis in Section 4.3, Agent 1 chooses x_{1H} to solve the following problem:

$$\max_{x_{1H}} \left(\frac{x_{1H}}{x_{1H} + x_{2H}} \right) \delta_{1H} v_H - x_{1H},$$

given that Agent 2 also chooses δ_{2H}. Then the equilibrium effort levels in the contest stage are $x_{1H}^* = \delta_{1H}^2 \delta_{2H} v_H / (\delta_{1H} + \delta_{2H})^2$ and $x_{2H}^* = \delta_{1H} \delta_{2H}^2 v_H / (\delta_{1H} + \delta_{2H})^2$. It can be show that Agent 1 obtains $\pi_1(\delta_{1H}; \delta_{2H}, v_H) = \delta_{1H}^3 v_H / (\delta_{1H} + \delta_{2H})^2$ after substituting the equilibrium efforts.

On the other hand, if Agent 1 instead chooses δ_{1L} (i.e., he pretends to be a L type), given that Agent 2 still chooses δ_{2H}, Agent 1 solves:

$$\max_{x_1} \left(\frac{x_1}{x_1 + x_{2H}} \right) \delta_{1L} v_H - x_1.$$

The effort levels chosen by the agents under this "deviation" are $x_1' = \delta_{1L}^2\delta_{2H}v_H / (\delta_{1L}+\delta_{2H})^2$ and $x_{2H}' = \delta_{1L}\delta_{2H}^2 v_H / (\delta_{1L}+\delta_{2H})^2$. In this case, the deviation payoff for Agent 1 is $\pi_1(\delta_{1L};\delta_{2H},v_H) = \delta_{1L}^3 v_H / (\delta_{1L}+\delta_{2H})^2$. Then Agent 1 will choose δ_{1H} when $\theta = H$ if

$$\begin{aligned}\pi_1(\delta_{1H};\delta_{2H},v_H) &= \frac{\delta_{1H}^3 v_H}{(\delta_{1H}+\delta_{2H})^2} \geq \pi_1(\delta_{1L};\delta_{2H},v_H) \\ &= \frac{\delta_{1L}^3 v_H}{(\delta_{1L}+\delta_{2H})^2} \quad \text{iff} \quad \delta_{1H} \geq \delta_{1L}.\end{aligned} \tag{A1}$$

By using the same logic, when Agent 1 observes $\theta = L$ and chooses the contract δ_{1L}, he obtains $\pi_1(\delta_{1L};\delta_{2L},v_L) = \delta_{1L}^3 v_L / (\delta_{1L}+\delta_{2L})^2$. On the other hand, if he instead chooses δ_{1H} (i.e., he pretends to be a H type), given that Agent 2 still truthfully chooses δ_{2L}, the deviation payoff for Agent 1 is $\pi_1(\delta_{1H};\delta_{2L},v_L) = \delta_{1H}^3 v_L / (\delta_{1H}+\delta_{2L})^2$. Thus, Agent 1 will choose δ_{1L} when $\theta = L$ if

$$\begin{aligned}\pi_1(\delta_{1L};\delta_{2L},v_L) &= \frac{\delta_{1L}^3 v_L}{(\delta_{1L}+\delta_{2L})^2} \geq \pi_1(\delta_{1H};\delta_{2L},v_L) \\ &= \frac{\delta_{1H}^3 v_L}{(\delta_{1H}+\delta_{2L})^2} \quad \text{iff} \quad \delta_{1L} \geq \delta_{1H}.\end{aligned} \tag{A2}$$

We can see that the only possibility for both (A1) and (A2) to hold is $\delta_{1H} = \delta_{1L}$. Thus, if $\delta_{1H} \neq \delta_{1L}$, one type must have the incentive to mimic the other, so that separation is impossible. In other words, both types will end up choosing the same contract (with the larger δ) so that the principal cannot distinguish them, and so there is no way to extract the private information.

□

Proof of Proposition 3.

We consider three separate cases:

Case 1: $v_H < 3v_L$. The payoff matrix is shown in Table A1.

Table A1 The Payoff Matrix in Stage 1 When $v_H < 3v_L$.

	$a_2 = N$	$a_2 = S$	$a_2 = D$
$a_1 = N$	$\frac{v_H+v_L}{8}, \frac{v_H+v_L}{8}$	$\frac{v_H+v_L}{16}+\frac{\sqrt{v_H v_L}}{8}$, $\frac{5(v_H+v_L)}{16}-\frac{3\sqrt{v_H v_L}}{8}-k$	$\frac{v_H+v_L}{4}$, $\frac{(2\sqrt{v_H+v_L}-\sqrt{v_H}-\sqrt{v_L})^2}{4}$
$a_1 = S$	$\frac{5(v_H+v_L)}{16}-\frac{3\sqrt{v_H v_L}}{8}-k$, $\frac{v_H+v_L}{16}+\frac{\sqrt{v_H v_L}}{8}$	$\frac{v_H+v_L}{8}-k, \frac{v_H+v_L}{8}-k$	$\frac{v_H+v_L}{4}-k$, $\frac{(\sqrt{2}-1)^2(v_H+v_L)}{2}$
$a_1 = D$	$\frac{(2\sqrt{v_H+v_L}-\sqrt{v_H}-\sqrt{v_L})^2}{4}$, $\frac{v_H+v_L}{4}$	$\frac{(\sqrt{2}-1)^2(v_H+v_L)}{2}$, $\frac{v_H+v_L}{4}-k$	$\frac{v_H+v_L}{6}, \frac{v_H+v_L}{6}$

Equilibrium (a): $(a_1^*, a_2^*) = (N, N)$.

To support this strategy profile as a Nash equilibrium, $a_1 = N$ should be a best response with respect to $a_2 = N$, which requires:

$$\frac{v_H+v_L}{8} \geq \frac{5(v_H+v_L)}{16} - \frac{3\sqrt{v_H v_L}}{8} - k \quad \text{or} \quad k \geq \frac{3(\sqrt{v_H}-\sqrt{v_L})^2}{16}, \tag{A3}$$

and

$$\frac{v_H+v_L}{8} \geq \frac{\left[2\sqrt{v_H+v_L}-(\sqrt{v_H}+\sqrt{v_L})\right]^2}{4}. \tag{A4}$$

However, since (A4) always holds in this range, it is redundant.

Similarly, $a_2 = N$ should be a best response with respect to $a_1 = N$. The Principal 2's problem is symmetric to that faced by Principal 1 and so the condition is the same. Therefore, $(a_1, a_2) = (N, N)$ can be a Nash equilibrium if (A3) holds.

Equilibrium (b): $(a_i^*, a_j^*) = (S, N)$.

We discuss the case $(a_1^*, a_2^*) = (S, N)$ here, and the other case $(a_1^*, a_2^*) = (N, S)$ is analogous. To support this strategy profile as a Nash equilibrium, $a_1 = S$ should be a best response with respect to $a_2 = N$, which requires:

$$\frac{5(v_H + v_L)}{16} - \frac{3\sqrt{v_H v_L}}{8} - k \geq \frac{v_H + v_L}{8} \quad \text{or} \quad k \leq \frac{3(\sqrt{v_H} - \sqrt{v_L})^2}{16}, \tag{A5}$$

and

$$\frac{5(v_H + v_L)}{16} - \frac{3\sqrt{v_H v_L}}{8} - k \geq \frac{\left[2\sqrt{v_H + v_L} - (\sqrt{v_H} + \sqrt{v_L})\right]^2}{4}. \tag{A6}$$

However, because (A4) always holds in this range, (A6) is implied by (A5) and thus is redundant.

On the other hand, $a_2 = N$ should be a best response with respect to $a_1 = S$. It requires

$$\frac{v_H + v_L}{16} + \frac{\sqrt{v_H v_L}}{8} \geq \frac{v_H + v_L}{8} - k \quad \text{or} \quad k \geq \frac{(\sqrt{v_H} - \sqrt{v_L})^2}{16}. \tag{A7}$$

and

$$\frac{v_H + v_L}{16} + \frac{\sqrt{v_H v_L}}{8} \geq \frac{(\sqrt{2} - 1)^2 (v_H + v_L)}{2}. \tag{A8}$$

However, since (A8) always holds in this range, it is redundant. Therefore, for $(a_1, a_2) = (S, N)$ (and $(a_1, a_2) = (N, S)$ too) to be supported as a Nash equilibrium, it requires (A5) and (A7) to hold, i.e., $(\sqrt{v_H} - \sqrt{v_L})^2 / 16 \leq k \leq 3(\sqrt{v_H} - \sqrt{v_L})^2 / 16$.

Equilibrium (c): $(a_1^*, a_2^*) = (S, S)$.

To support this strategy profile as a Nash equilibrium, $a_1 = S$ should be a best response with respect to $a_2 = S$, which requires:

$$\frac{v_H + v_L}{8} - k \geq \frac{v_H + v_L}{16} + \frac{\sqrt{v_H v_L}}{8} \quad \text{or} \quad k \leq \frac{(\sqrt{v_H} - \sqrt{v_L})^2}{16}, \tag{A9}$$

and

$$\frac{v_H + v_L}{8} - k \geq \frac{(\sqrt{2} - 1)^2 (v_H + v_L)}{2}. \tag{A10}$$

Again, because (A8) always holds in this range, (A10) is implied by (A9) and thus is redundant.

Similarly, $a_2 = S$ should be a best response with respect to $a_1 = S$. The Principal 2's problem is symmetric to that faced by Principal 1 and thus the condition is the same. Therefore, $(a_1, a_2) = (S, S)$ can be a Nash equilibrium if (A9) holds.

Case 2: $3v_L \leq v_H < 9v_L$. The payoff matrix is shown in Table A2. The only difference between Table A1 and Table A2 is the case $(a_1, a_2) = (D, N)$ and (N, D). However, there is no fundamental difference in the analysis and the equilibrium outcomes are the same as those in the previous Case 1. Therefore, this case is skipped.

Case 3: $v_H \geq 9v_L$. The Payoff Matrix is shown in Table A3.

Equilibrium (a): $(a_1^*, a_2^*) = (N, N)$.

To support this strategy profile as a Nash equilibrium, $a_1 = N$ should be a best response with respect to $a_2 = N$, which requires:

$$\frac{v_H + v_L}{8} \geq \frac{2v_H}{9} - k \quad \text{or} \quad k \geq \frac{7v_H}{72} - \frac{v_L}{8}, \tag{A11}$$

Table A2 The Payoff Matrix in Stage 1 When $3v_L \le v_H < 9v_L$.

	$a_2 = N$	$a_2 = S$	$a_2 = D$
$a_1 = N$	$\frac{v_H + v_L}{8}, \frac{v_H + v_L}{8}$	$\frac{v_H + v_L}{16} + \frac{\sqrt{v_H v_L}}{8}$, $\frac{5(v_H + v_L)}{16} - \frac{3\sqrt{v_H v_L}}{8} - k$	$\frac{v_H}{6} + \frac{v_L}{2}, \frac{(\sqrt{3}-1)^2 v_H}{4}$
$a_1 = S$	$\frac{5(v_H + v_L)}{16} - \frac{3\sqrt{v_H v_L}}{8} - k$, $\frac{v_H + v_L}{16} + \frac{\sqrt{v_H v_L}}{8}$	$\frac{v_H + v_L}{8} - k, \frac{v_H + v_L}{8} - k$	$\frac{v_H + v_L}{4} - k$, $\frac{(\sqrt{2}-1)^2 (v_H + v_L)}{2}$
$a_1 = D$	$\frac{(\sqrt{3}-1)^2 v_H}{4}, \frac{v_H}{6} + \frac{v_L}{2}$	$\frac{(\sqrt{2}-1)^2 (v_H + v_L)}{2}$, $\frac{v_H + v_L}{4} - k$	$\frac{v_H + v_L}{6}, \frac{v_H + v_L}{6}$

Table A3 The Payoff Matrix in Stage 1 When $v_H < 9v_L$.

	$a_2 = N$	$a_2 = S$	$a_2 = D$
$a_1 = N$	$\frac{v_H + v_L}{8}, \frac{v_H + v_L}{8}$	$\frac{v_H}{18} + \frac{v_L}{2}, \frac{2v_H}{9} - k$	$\frac{v_H}{6} + \frac{v_L}{2}, \frac{(\sqrt{3}-1)^2 v_H}{4}$
$a_1 = S$	$\frac{2v_H}{9} - k, \frac{v_H}{18} + \frac{v_L}{2}$	$\frac{v_H + v_L}{8} - k, \frac{v_H + v_L}{8} - k$	$\frac{v_H + v_L}{4} - k$, $\frac{(\sqrt{2}-1)^2 (v_H + v_L)}{2}$
$a_1 = D$	$\frac{(\sqrt{3}-1)^2 v_H}{4}, \frac{v_H}{6} + \frac{v_L}{2}$	$\frac{(\sqrt{2}-1)^2 (v_H + v_L)}{2}$, $\frac{v_H + v_L}{4} - k$	$\frac{v_H + v_L}{6}, \frac{v_H + v_L}{6}$

and

$$\frac{v_H + v_L}{8} \ge \frac{(\sqrt{3}-1)^2 v_H}{4} \quad \text{or} \quad v_H \le \frac{v_L}{7 - 4\sqrt{3}}. \tag{A12}$$

Similarly, $a_2 = N$ should be a best response with respect to $a_1 = N$. The Principal 2's problem is symmetric to that faced by Principal 1 and thus the conditions are the same. Therefore, $(a_1, a_2) = (N, N)$ can be a Nash equilibrium if (A11) and (A12) hold.

Equilibrium (b): $(a_i^*, a_j^*) = (S, N)$.

We discuss the case $(a_1^*, a_2^*) = (S, N)$ here, and the other case $(a_1^*, a_2^*) = (N, S)$ is analogous. To support this strategy profile as a Nash equilibrium, $a_1 = S$ should be a best response with respect to $a_2 = N$, which requires:

$$\frac{2v_H}{9} - k \geq \frac{v_H + v_L}{8} \quad \text{or} \quad k \leq \frac{7v_H}{72} - \frac{v_L}{8}, \tag{A13}$$

and

$$\frac{2v_H}{9} - k \geq \frac{(\sqrt{3}-1)^2 v_H}{4} \quad \text{or} \quad k \leq \frac{(9\sqrt{3}-14)v_H}{18}. \tag{A14}$$

Note that $(7v_H / 72) - (v_L / 8) < (9\sqrt{3} - 14)v_H / 18$ if and only if $v_H < v_L / (7 - 4\sqrt{3})$.

On the other hand, $a_2 = N$ should be a best response with respect to $a_1 = S$. It requires

$$\frac{v_H}{18} + \frac{v_L}{2} \geq \frac{v_H + v_L}{8} - k \quad \text{or} \quad k \geq \frac{5v_H}{72} - \frac{3v_L}{8}. \tag{A15}$$

and

$$\frac{v_H}{18} + \frac{v_L}{2} \geq \frac{(\sqrt{2}-1)^2 (v_H + v_L)}{2} \quad \text{or} \quad v_H \leq \frac{9(\sqrt{2}-1)v_L}{13 - 9\sqrt{2}}. \tag{A16}$$

We note that since $9(\sqrt{2}-1)/(13-9\sqrt{2}) < 1/(7-4\sqrt{3})$, we have $7v_H / 72 - v_L /$

$8 < (9\sqrt{3}-14)v_H/18$, and so (A14) is implied by (A13) and thus is redundant. Therefore, for $(a_1, a_2) = (S, N)$ (and (N, S) too) to be supported as a Nash equilibrium, it requires (A13), (A15), and (A16), i.e., $5v_H/72 - 3v_L/8 \le k \le (7v_H/72) - (v_L/8)$ and $v_H \le 9(\sqrt{2}-1)v_L/(13-9\sqrt{2})$.

Equilibrium (c): $(a_1^*, a_2^*) = (S, S)$.

To support this strategy profile as a Nash equilibrium, $a_1 = S$ should be a best response with respect to $a_2 = S$, which requires:

$$\frac{v_H + v_L}{8} - k \ge \frac{v_H}{18} + \frac{v_L}{2} \quad \text{or} \quad k \le \frac{5v_H}{72} - \frac{3v_L}{8}, \tag{A17}$$

and

$$\frac{v_H + v_L}{8} - k \ge \frac{(\sqrt{2}-1)^2(v_H + v_L)}{2} \quad \text{or} \quad k \le \frac{(8\sqrt{2}-11)(v_H + v_L)}{8}. \tag{A18}$$

Similarly, $a_2 = S$ should be a best response with respect to $a_1 = S$. The Principal 2's problem is symmetric to that faced by Principal 1 and thus the conditions are the same. Therefore, $(a_1, a_2) = (S, S)$ can be a Nash equilibrium if both (A17) and (A18) hold, i.e., $k \le \min[5v_H/72 - 3v_L/8, (8\sqrt{2}-11)(v_H + v_L)/8]$.

Equilibrium (d): $(a_i^*, a_j^*) = (D, N)$.

We discuss the case $(a_1^*, a_2^*) = (D, N)$ here, and the other case $(a_i^*, a_j^*) = (N, D)$ is analogous. To support this strategy profile as a Nash equilibrium, $a_1 = D$ should be a best response with respect to $a_2 = N$, which requires:

$$\frac{(\sqrt{3}-1)^2 v_H}{4} \ge \frac{v_H + v_L}{8} \quad \text{or} \quad v_H \ge \frac{v_L}{7-4\sqrt{3}}, \tag{A19}$$

and

$$\frac{(\sqrt{3}-1)^2 v_H}{4} \geq \frac{2v_H}{9} - k \quad \text{or} \quad k \geq \frac{(9\sqrt{3}-14)v_H}{18}. \tag{A20}$$

On the other hand, $a_2 = N$ should be a best response with respect to $a_1 = D$. It requires

$$\frac{v_H}{6} + \frac{v_L}{2} \geq \frac{v_H + v_L}{4} - k \quad \text{or} \quad k \geq \frac{v_H}{12} - \frac{v_L}{4}. \tag{A21}$$

and

$$\frac{v_H}{6} + \frac{v_L}{2} \geq \frac{v_H + v_L}{6}. \tag{A22}$$

Since (A22) always holds, it is redundant. Moreover, $(9\sqrt{3}-14)v_H/18 > v_H/12 - v_L/4$, so (A21) is implied by (A20) and thus is redundant. Thus, (a_1, a_2) = (D, N) (and (N, D) too) can be supported as a Nash equilibrium if both (A19) and (A20) hold.

Equilibrium (e): $(a_i^*, a_j^*) = (D, S)$.

We discuss the case $(a_1^*, a_2^*) = (S, D)$ here, and the other case $(a_1^*, a_2^*) = (D, S)$ is analogous. To support this strategy profile as a Nash equilibrium, $a_1 = S$ should be a best response with respect to $a_2 = D$, which requires:

$$\frac{v_H + v_L}{4} - k \geq \frac{v_H}{6} + \frac{v_L}{2} \quad \text{or} \quad k \leq \frac{v_H}{12} - \frac{v_L}{4}. \tag{A23}$$

and

$$\frac{v_H + v_L}{4} - k \geq \frac{v_H + v_L}{6}. \tag{A24}$$

Since (A22) always holds, (A24) is implied by (A23) and thus it is redundant.

On the other hand, $a_2 = D$ should be a best response with respect to $a_1 = S$. It requires

$$\frac{(\sqrt{2}-1)^2(v_H+v_L)}{2} \geq \frac{v_H}{18}+\frac{v_L}{2} \quad \text{or} \quad v_H \geq \frac{9(\sqrt{2}-1)v_L}{13-9\sqrt{2}}, \tag{A25}$$

and

$$\frac{(\sqrt{2}-1)^2(v_H+v_L)}{2} \geq \frac{v_H+v_L}{8}-k \quad \text{or} \quad k \geq \frac{(8\sqrt{2}-11)(v_H+v_L)}{8}. \tag{A26}$$

Thus, $(a_1, a_2) = (S, D)$ (and (D, S) too) can be supported as a Nash equilibrium if (A23), (A25) and (A26) hold, i.e., $(8\sqrt{2}-11)(v_H+v_L)/8 \leq k \leq v_H/12 - v_L/4$ and $v_H \geq 9(\sqrt{2}-1)v_L/(13-9\sqrt{2})$.

□

Table A4 The Social Welfare When $v_H < 3v_L$.

	$a_2 = N$	$a_2 = S$	$a_2 = D$
$a_1 = N$	$\frac{v_H+v_L}{4}$	$\frac{3(v_H+v_L)}{8} - \frac{\sqrt{v_H v_L}}{4} - k$	$\frac{v_H+v_L}{2} - \frac{(\sqrt{v_H}+\sqrt{v_L})(\sqrt{v_H}+\sqrt{v_L}-\sqrt{v_H+v_L})}{4}$
$a_1 = S$	$\frac{3(v_H+v_L)}{8} - \frac{\sqrt{v_H v_L})}{4} - k$	$\frac{v_H+v_L}{4} - 2k$	$\frac{\sqrt{2}(v_H+v_L)}{4} - k$
$a_1 = D$	$\frac{v_H+v_L}{2} - \frac{(\sqrt{v_H}+\sqrt{v_L})(\sqrt{v_H}+\sqrt{v_L}-\sqrt{v_H+v_L})}{4}$	$\frac{\sqrt{2}(v_H+v_L)}{4} - k$	$\frac{5(v_H+v_L)}{12}$

Table A5 The Social Welfare When $3v_L \le v_H < 9v_L$.

	$a_2 = N$	$a_2 = S$	$a_2 = D$
$a_1 = N$	$\frac{v_H + v_L}{4}$	$\frac{3(v_H + v_L)}{8} - \frac{\sqrt{v_H v_L}}{4} - k$	$\frac{v_H}{3} + \frac{v_L}{2}$
$a_1 = S$	$\frac{3(v_H + v_L)}{8} - \frac{\sqrt{v_H v_L}}{4} - k$	$\frac{v_H + v_L}{4} - 2k$	$\frac{\sqrt{2}(v_H + v_L)}{4} - k$
$a_1 = D$	$\frac{v_H}{3} + \frac{v_L}{2}$	$\frac{\sqrt{2}(v_H + v_L)}{4} - k$	$\frac{5(v_H + v_L)}{12}$

Table A6 The Social Welfare When $v_H \ge 9v_L$.

	$a_2 = N$	$a_2 = S$	$a_2 = D$
$a_1 = N$	$\frac{v_H + v_L}{4}$	$\frac{5v_H}{18} + \frac{v_L}{2} - k$	$\frac{v_H}{3} + \frac{v_L}{2}$
$a_1 = S$	$\frac{5v_H}{18} + \frac{v_L}{2} - k$	$\frac{v_H + v_L}{4} - 2k$	$\frac{\sqrt{2}(v_H + v_L)}{4} - k$
$a_1 = D$	$\frac{v_H}{3} + \frac{v_L}{2}$	$\frac{\sqrt{2}(v_H + v_L)}{4} - k$	$\frac{5(v_H + v_L)}{12}$

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摘　要

本文考慮兩團隊競逐一個固定價值獎品之雇傭代理模型。 此獎品之真正價值對兩隊都相同，但只有兩隊之代理人才知道。兩隊之委託人則只知道其值可能為高或低，並不知道其真正價值。因此，兩隊之委託人有兩種可能策略：「親為」是指委託人親自下海付出一定成本以獲得此標的獎品之價值資訊，而「授權」則是授權給代理人出面為其競爭此獎品。我們發現，當資訊獲取成本和獎品高低值差距皆很大時，只有一個委託人會採取「授權」的方式。然而，從社會福利的角度，兩人都「授權」才是較佳的選擇。

關鍵詞：尋租競賽、訊息獲取、授權
JEL分類代號：C72, D81, D82

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投稿日期：110 年 5 月 7 日；修訂日期：110 年 7 月 7 日；
接受日期：111 年 5 月 27 日。